Thermal Duality Transformations and the Canonical Ensemble
The Deconfining Long String Phase Transition

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Abstract

We give a first principles formulation of the equilibrium statistical mechanics of strings in the canonical ensemble, compatible with the Euclidean timelike T-duality transformations that link the six supersymmetric string theories in pairs: heterotic $E_8{\times}E_8$ and $\mathrm{Spin}(32)/\mathrm{Z}_2$, type IIA and type IIB, or type IB and type I′. We demonstrate that each of the supersymmetric string ensembles exhibits a T^2 growth in the free energy at high temperatures far above the string scale, precisely as conjectured by Atick and Witten in 1988, and shown to follow as a consequence of thermal self-duality in the closed bosonic string ensemble by Polchinski more recently. We verify that the low energy field theory limit of our expression for the string free energy reproduces the expected T^{10} growth when the contribution from massive string modes is suppressed. In every case, heterotic, type I, and type II, we can definitively rule out the occurrence of an exponential divergence in the one-loop string free energy above some critical temperature: *previous claims of evidence for the breakdown of the supersymmetric string canonical ensembles at a limiting Hagedorn temperature are incorrect.* Finally, we identify a macroscopic loop amplitude in the type I string theories which yields the expectation value of a single Wilson-Polyakov-Susskind loop in the low energy finite temperature supersymmetric gauge theory limit, an order parameter for a thermal phase transition at a string scale temperature. We point out that precise computations can nevertheless be carried out on either side of the phase boundary by using the low energy finite temperature supersymmetric gauge theory limits of the pair of thermal dual string theories, type IB and type I′. Note Added (Sep 2005).

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1   Introduction

T-duality invariance, the result of interchanging small radius with large radius, $R \to \alpha'/R$, is a *spontaneously* broken symmetry in String/M Theory: in other words, a T-duality transformation on an embedding target space coordinate will, in general, map a given background of String/M theory to a different background of the same theory. To be specific, the circle-compactified $E_8 \times E_8$ heterotic oriented closed string theory is mapped under a T-duality to the circle-compactified $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic oriented closed string theory [15, 17, 18, 49, 50, 40]. The circle-compactified type IIA oriented closed string theory is mapped to the circle-compactified type IIB oriented closed string theory [16, 19]. And, finally, the unoriented type IB open and closed string theory is mapped under a T-duality transformation to the, rather unusual, type I$'$ unoriented open and closed string theory: the integer-moded open string momentum modes associated with the compact coordinate are mapped to the integer-moded closed string winding modes in the T-dual type I$'$ theory [19, 69, 32].

In this paper we will examine the consequences of such T-duality transformations on the Wick-rotated time coordinate X^0. It is clear that a Wick rotation on X^0 maps the noncompact $SO(9,1)$ Lorentz invariant background of a given supersymmetric string theory to the corresponding $SO(10)$ invariant background, with an embedding time coordinate of Euclidean signature. The Wick-rotated $SO(10)$ invariant background arises naturally in any formulation of equilibrium string statistical mechanics in the canonical ensemble, the statistical ensemble characterized by fixed temperature and fixed spatial volume (β, V). The Polyakov path integral over connected world-surfaces is formulated in a target spacetime of fixed spacetime volume. Thus, the one-loop vacuum functional in the $SO(10)$ invariant background computes precisely the sum over connected one-loop vacuum graphs in the target space R^9 in the finite temperature vacuum at temperature T [1]. The appearance of a *tachyonic* mode in the string thermal spectrum is an indication that the worldsheet conformal field theory is no longer at a fixed point of the 2d Renormalization Group (RG): the tachyon indicates a relevant flow of the 2d RG. The question of significance is then as follows: does the relevant flow terminate in a new infrared fixed point? If so, the new fixed point determines the true thermal string vacuum. An equilibrium statistical mechanics of strings requires that this fixed point belong to a *fixed line* parameterized by inverse temperature β [36, 5]: the precise analog under Wick rotation of the line of fixed points parameterized by the radius of a compact spatial coordinate in the $SO(9,1)$ vacuum.

Target spacetime supersymmetry, and its spontaneous breaking in the thermal vacuum along the line of fixed points parameterized by β, introduces new features into this discussion. We must require compatibility with the expected properties of the low energy field theory limit where the contribution from massive string modes has been suppressed, namely, those of a 10D finite temperature supersymmetric gauge theory. We must also require consistency with string theoretic symmetries of both worldsheet, and target space, origin in the Wick rotated $SO(10)$ invariant background. In particular, Euclidean T-duality transformations must link the thermal vacua of the six different supersymmetric string theories in pairs: heterotic $E_8 \times E_8$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$, type IIA and type IIB, and type IB and type I$'$. Remarkably, we will find as a direct consequence of the T-duality transformations that the tachyonic thermal instabilities arising in all previous attempts

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to formulate an equilibrium supersymmetric string statistical mechanics in the canonical ensemble are simply *absent*.

The earliest works on the statistical mechanics of strings focussed on the density of states function for the superconformal field theory (SCFT) on a strip [12, 20, 62], a calculation that enters into the one-loop vacuum amplitude of an open string theory. The degeneracy, $P(N)$, at level N, with masses scaling as $(N/\alpha')^{1/2}$, grows rapidly with increasing level in any 2d SCFT. Thus, the asymptotic behavior of the density of states function at high oscillator levels displays an exponential divergence characterized by the total central charge, c, of the SCFT [12, 20, 62]. This result follows from the famous Hardy-Ramanujan formula [13]:

$$\prod_{k=1}^{\infty}(1-q^k)^{-c} = \sum_N P(N;c) \; q^N, \quad \lim_{N\to\infty} P(N;c) = N^{-(c+3)/4}\exp\left[2\pi(cN/6)^{1/2}\right] \quad , \qquad (1)$$

where $q=e^{-2\pi t}$, and t is the edge-length of the strip. The coefficient in the exponential has become known in the string theory literature as the Hagedorn temperature associated with the superconformal field theory of central charge c:

$$T_H = 1/2\pi(c/6)^{1/2}\alpha'^{1/2} \quad . \qquad (2)$$

While this is an interesting property of the central charge of the SCFT including the contribution from both worldsheet bosonic, and fermionic, modes, notice that this argument has not take into account the physical state conditions of the supersymmetric *string* theory: these are couched in the form of a projection on the Hilbert space of the SCFT. The partition function of the SCFT on a strip is much more than the sum over an infinite set of free field oscillator contributions of total central charge c accounted for by the Hardy-Ramanujan formula given above: it takes the form of a sum over integer powers of the Jacobi theta functions, and each such term contributes to the partition function with a *phase* determined by the physical state projection on the Hilbert space of the SCFT. The choice of phases, which can always be interpreted in terms of the choice of *spin structure* for the worldsheet fermions on the annulus, and which are modified on the nonorientable Riemann surfaces arising in the presence of an orientifold plane in the embedding target spacetime as shown in sections 3 and 4, is therefore heavily constrained by both worldsheet, and *target space*, symmetries.

As a consequence of the phases in the partition function on the strip, an exponential divergence in the ultraviolet asymptotic density of states function for a SCFT of given central charge c does *not* necessarily signify a divergence in the vacuum energy density for the finite temperature string vacuum. A generic property of all supersymmetric string amplitudes, whether unoriented open and closed, or orientable closed [31, 1, 8, 32], is the fact that worldsheet divergences originating in the ultraviolet asymptotics of the SCFT can always be mapped to an equivalent divergence of infrared origin: in the open and closed strings, this is a consequence of open-closed channel duality which maps the ultraviolet behavior of the closed string sector to the infrared of the open string sector, and vice versa. For the orientable closed superstrings: orientable type IIA, IIB, and heterotic, this property follows as a consequence of the invariance of closed string amplitudes under the modular group of the Riemann surface. Modular invariance can be invoked to excise the domain of the integral over moduli that would be dominated by contributions from the deep ultraviolet regime of

the closed string spectrum. In other words, although the Hardy-Ramanujan exponential divergence in the ultraviolet asymptotics of the free field oscillator sums in the 2d SCFT is a fact, it is *not* necessarily indicative of a divergence in the energy density of the string thermal vacuum: the important question is whether there are any uncancelled *infrared* divergences in the string vacuum amplitude? This requires analysis of the infrared asymptotics of the vacuum amplitude, separating the individual contributions from the tachyonic, and massless, modes that dominate the infrared behavior of the amplitudes. These correspond, respectively, to the relevant, and marginal, operators in the physical Hilbert space of the SCFT.

The reader will be familiar with the famous GSO projection which eliminates the zero temperature tachyon from the physical spectrum of the supersymmetric string theories [32]. The thermal physical state conditions we will invoke in what follows achieve a similar projection for all of the *tachyonic* thermal modes in the worldsheet SCFT. With all of the thermal tachyons projected out of the physical Hilbert space of the SCFT, and upon verifying the cancellation of massless tadpoles in the factorization limit of the vacuum amplitude, we will have succeeded in eliminating all sources of divergence in the vacuum energy density. It is one purpose of this paper to explain that this procedure can be successfully carried out for each of the six supersymmetric string theories. We will find that each has a viable thermal vacuum in which we can formulate an equilibrium string statistical mechanics in the canonical ensemble. In the case of the type IIA and type IIB superstring theories, we will find that a viable thermal vacuum *requires* the introduction of Dbranes carrying Ramond-Ramond charge, and consequently, an open string sector with Yang-Mills gauge fields in the low energy limit.

Having clarified that the canonical ensemble of supersymmetric strings is well-defined at all temperatures, including the temperature regime far above the string mass scale, $\alpha'^{-1/2}$, we will establish several new results in this paper. First, we show that in all of the string ensembles: heterotic, or type I and type II, the growth of the vacuum energy density, $\rho(\beta)$, at high temperatures far beyond the string scale is only as fast as in a 2d quantum field theory. Thus, as was conjectured with only limited intuition as far back as 1988 by Atick and Witten [2], there is a dramatic reduction in the *growth* of the vacuum energy density at high temperatures. More recently [32], Polchinski has shown that the T^2 growth in the vacuum energy density at high temperatures is a direct consequence of the thermal self-duality of the vacuum functional in the closed bosonic string theory. We will show in sections 3.2, and 5.1, respectively, that there exists a precise analog of this behavior for all six supersymmetric string theories: in each case, the T^2 growth in the vacuum energy density at high temperatures follows as a simple consequence of the Euclidean T-duality *transformations* that link the thermal ground states of the supersymmetric string theories in pairs.

The type IB, and type I$'$, unoriented open and closed string ensembles are especially interesting because of the close comparison that can be made with the finite temperature supersymmetric Yang-Mills gauge theory obtained in the low energy limit, where the contribution to the vacuum amplitude from massive string modes has been suppressed. In particular, we verify in section 4.3 that the characteristic, stringy, T^2 high temperature growth described above is compatible with the much faster T^{10} growth at temperatures far below the string scale, as expected by comparison with finite temperature quantum field theory. The fact that the energy density grows much more *slowly* at high temperatures beyond the string scale is a hint indicating a likely thermal phase transition.

Our strategy in section 5 for exposing this phase transition is as follows: since the string vacuum amplitude has shown no sign of a discontinuity, or non-analyticity, as a function of temperature we must look at a different amplitude, or correlation function, as a plausible order parameter for a thermal phase transition. A natural choice suggested by the correspondence in the low energy limit to a finite temperature Yang-Mills gauge theory, would be the string theory analog of the expectation value of a timelike Wilson-Polyakov-Susskind loop wrapping the Euclidean time direction, namely, the change in the free energy in the thermal vacuum due to the introduction of an external heavy quark, generally taken to be the order parameter for the deconfinement phase transition in finite temperature gauge theory [64, 56]. We should note that this quantity is extremely sensitive to infrared divergences in finite temperature gauge theory, necessitating ingenious techniques for a clear-cut study of the order parameter in both the lattice, or dual confinement model, approaches [65]. It is generally considered easier in 4d gauge theory to extract the desired result from a computation of the *pair correlator* of Polyakov-Susskind loops which yields the static heavy quark-antiquark potential in the thermal vacuum.

In string theory, it turns out that the Polyakov path integral summing surfaces with the topology of an annulus and with boundaries mapped to a pair of fixed curves, $\mathcal{C}_1$, $\mathcal{C}_2$, in the embedding target spacetime, wrapping the Euclidean time coordinate, and with fixed spatial separation, R, can also be computed from first principles using Riemann surface methodology. This computation is an *extension* of the one-loop vacuum amplitude computation due to Polchinski [1]. The amplitude can be interpreted as an off-shell closed string tree propagator, and the result in closed bosonic string theory, but only in the limit that the macroscopic boundaries, $\mathcal{C}_1$, $\mathcal{C}_2$, were point-like, was first obtained by Cohen, Moore, Nelson, and Polchinski [72]. Their 1986 analysis was recently completed by myself in collaboration with Yujun Chen and Eric Novak [73] including, in particular, the limit of *large* macroscopic loop length of interest to us here. The extension to the macroscopic loop amplitude in the type I and type II string theories with Dbranes appears in [74].

Thus, in section 5.2, we will calculate the pair correlator of a pair of Polyakov-Susskind loops wrapping the Euclidean time coordinate, extracting the low energy gauge theory limit of the resulting expression where the contribution from massive string modes has been suppressed. Notice that in the limit of vanishing spatial separation, $R \to 0$, the amplitude will be dominated by the shortest open strings, namely, the gauge theory modes in the massless open string spectrum, and *the worldsheet collapses to a single macroscopic Wilson-Polyakov-Susskind loop wound around the Euclidean time coordinate*. We can analyze this limit of the expression for its dependence on temperature. We find clear evidence for a thermal phase transition in the gauge theory limit at a transition temperature of order the string mass scale.

The argument is as follows. A Euclidean T-duality transformation on our expression for the macroscopic loop amplitude in the type IB string theory conveniently maps it to an expression for a corresponding amplitude in the type I$'$ string theory. This expression will be well-defined in the temperature regime above T_C, and the low energy gauge theory limit is easily taken as before. Thus, the existence of T-dual type IB, and type I$'$, descriptions of the thermal ground state enable precise computations to be made in the low energy gauge theory limit on either side of the phase boundary. The intuition that a gas of short open strings transitions into a high temperature long string phase is an old piece of string folklore, dating to Hagedorn's 1965 paper [12, 25, 26, 27, 29, 77, 61, 62].

The outline of this paper is as follows. Section 2 gives an annotated overview of some significant previous developments in the statistical mechanics of strings, with primary focus on the worldsheet formulation for the canonical ensemble following Polchinski's analysis of the closed bosonic string ensemble in [1]. We clarify the precise differences between previous attempts, and our work. Section 3.1 contains the first-principles discussion of the tachyon-free heterotic string ensemble, emphasizing the significance of the Euclidean T-duality transformation that links the thermal ground states of the two heterotic string theories, and clarifying the low energy interpretation of the timelike Wilson line in terms of the axial gauge, $A_0 = 1/\beta$, quantization of the finite temperature gauge theory. In section 3.2, we demonstrating the T^2 growth in the vacuum energy density at temperatures above the string scale, and also establish the existence of the Kosterlitz-Thouless duality transition [30]. In section 4, we introduce the type I and type II unoriented open and closed string theories with Dbranes. Beginning with the closed orientable sector of these theories, we show in section 4.1 that the requirement of modular invariance is incompatible with the absence of thermal tachyons in the pure orientable closed string spectrum at temperatures *above* $T_{\text{w}=1}$, the temperature at which the first thermal winding mode crosses the threshold from irrelevance to marginal relevance. In section 4.2, we review the concept of renormalization group (RG) flows induced as a consequence of a relevant operator in the physical Hilbert space of the SCFT, from the perspective of both the target spacetime and worldsheet. We quote the result that follows from the 2d RG analysis in the accompanying paper [5], namely, that the integrable RG flow is in the direction towards the infrared fixed point corresponding to the noncompact, supersymmetric type II vacuum at zero temperature.

Section 4.3 describes the computation of the one-loop vacuum energy density at finite temperature in the remaining sectors of the type IB unoriented open and closed string theory. Remarkably, we find that summing the contributing worldsheets with annulus, Mobius Strip, and Klein Bottle topologies gives a *vanishing* one-loop vacuum energy density in the thermal type IB vacuum. This is despite the fact that target space supersymmetry has been broken by the thermal type IB physical state conditions. The vanishing of the one-loop vacuum energy density is a direct consequence of our requiring the absence of a tadpole for an unphysical Ramond-Ramond state, a consistency condition we must impose in the thermal vacuum for the same reasons it was imposed in the supersymmetric type IB vacuum [8, 69, 32, 5]. It is most intriguing to find a nonsupersymmetric type I vacuum, with supersymmetry broken by finite temperature effects, despite a vanishing one-loop vacuum energy density, and vanishing dilaton tadpole. Section 5.1 demonstrates the T^2 growth in the bosonic contributions to the type IB vacuum energy density, compatible with the T^{10} growth in the low energy limit when the contributions from massive string modes is suppressed. Section 5.2 summarizes our evidence for a thermal phase transition at the string scale in the low energy gauge theory limit of the type I string theory. Our conclusions, and discussion of some future directions suggested by our results, appear in section 6.

2   An Overview of the Statistical Mechanics of Strings

In this section, we provide an annotated overview of some significant developments in the statistical mechanics of strings. The development of this subject began almost forty years ago with Hagedorn's tantalizing suggestion in 1965 of a possible deconfining phase transition characterized by the ap-

pearance of a "long string" in bag models for hadronic physics [12]. We emphasize that this rough intuition predates the identification of QCD_3 as the correct theory of the strong interactions. It also predates the development of perturbative superstring theory as a unified theory encompassing both quantum gravity and the nonabelian gauge theories. It is interesting to note that the earliest motivations for the development of a statistical mechanics of strings arose in models for the non-perturbative sector of the strong interactions [12] and, subsequently, in models for cosmic string dynamics [26, 27]. Both are areas of considerable current research interest actively exploring the interface with String/M theory [55, 56, 59, 68, 75].

Our focus in this paper is on the canonical ensemble of strings at fixed (β, V), and its formulation in terms of the worldsheet formalism of perturbative string theory following the appearance of Polchinski's first-principles analysis of the closed bosonic string ensemble in [1]. We will clarify the precise differences in the extension of this first-principles approach to the supersymmetric string ensembles given in our work from some of the more significant attempts made by other authors.[2] A complementary review of the literature the reader may find useful can be found in [62]. Polchinski gave the original derivation of the free energy of the canonical ensemble of free closed bosonic strings starting with the Polyakov path integral summing connected Riemann surfaces in [1], correctly incorporating thermal self-duality of the closed bosonic string theory [32]. However, in physically interpreting the result for the free energy derived in [1], he omits to mention that there are low temperature tachyonic thermal momentum modes in the spectrum *at all temperatures starting from zero* [4]. Thus, Polchinski's oft-quoted identification of the first winding mode instability with the onset of a Hagedorn phase transition is suspect, an inference independently arrived at with the same incorrect reasoning by both Kogan [21], and by Sathiapalan [22]. As pointed out by us in [4, 5], the bosonic string free energy is already ridden with tachyonic divergences long before the so-called transition temperature has been reached: there is no self-consistent equilibrium ensemble of closed bosonic strings [4]. But the methodology formulated for this special case by Polchinski in [1] can be extended with success in the case of all six *supersymmetric* string theories, as was pointed out by us in [3], and in this current work.

Returning to our account of relevant previous developments following the appearance of [1], a proposal for the one-loop free energy of the heterotic string canonical ensemble was subsequently given by O'Brien and Tan [24]: the expression contains tachyonic divergences and is untenable as a proposal for equilibrium statistical mechanics. Related discussions of Hagedorn divergences, and of duality in the thermal mode spectrum, appear in the works [15, 20, 23]. The calculation in [24] did not take into account a crucial consequence of thermal duality in the heterotic string: namely, that the $E_8 \times E_8$ and $\text{Spin}(32)/Z_2$ supersymmetric ground states at, respectively, the zero, and infinite, temperature are interchanged under a thermal duality transformation on the finite temperature vacuum functional. The expression for the free energy consequently displays tachyonic thermal instabilities despite the fact that, unlike the closed bosonic string, the zero temperature spectrum of either heterotic string theory is tachyon-free. It is well-known that in the presence of a tachyonic mode the string ground state is no longer stable but is liable to evolve under renormalization group

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[2]Due to limitations of space, and given that most of the string literature repeats the misconceptions in both physical interpretation, or calculation, made in the standard treatments, we are unable to cite all of the relevant papers in this area.

flow to a different infrared-stable vacuum. Such a situation needs to be analyzed on a case-by-case basis to check the properties of the infrared-stable endpoint of renormalization group flow: *a string background with a tachyonic thermal mode is not in itself an acceptable setting for a discussion of equilibrium string thermodynamics.* Thus, a property we ought to require of the expression for the string vacuum functional at finite temperature is the *absence* of tachyonic thermal modes. As shown in our work, unlike the closed bosonic string theory which has a zero temperature tachyon [1], in the case of all six supersymmetric string theories it is indeed possible to arrive at a tachyon-free thermal spectrum.

Start with the heterotic string theory. How do we achieve a tachyon-free thermal spectrum, while simultaneously ensuring that the expression for the heterotic string vacuum functional satisfies the thermal duality relations as well as all of the worldsheet consistency conditions required of a string background, namely, super-Weyl and super-diffeomorphism invariance, including, in particular, invariance under modular transformations? The answer for the heterotic string [3] can, in fact, be deduced by piecing together results already existent in the string literature. In 1987, Ginsparg [40] solved an analogous technical problem, but without breaking supersymmetry, when he gave the general solution for the one-loop vacuum functional of the circle-compactified heterotic string theory on $R^9 \times S^1$, interpolating between the known results for the supersymmetric 10D $E_8 \times E_8$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$ strings in the limit of infinite and zero radius. The reader should keep in mind the correspondence between compactification radius and inverse temperature: $R \leftrightarrow \beta/2\pi$. Notice that the nonabelian gauge symmetry at generic radius found in [40] was only $SO(16) \times SO(16)$: this is an inviolable consequence of T-duality, which interchanges heterotic string ground states with different nonabelian gauge group at zero and infinite radius. The interpolation is achieved by turning on a Wilson line in the nonabelian gauge theory which is radius dependent, using the Lorentzian self-dual lattice parametrization of generic heterotic string backgrounds developed in [39]. We now recall another result from the literature, namely, that there is a *unique* heterotic modular invariant choice of spin structures in nine or ten dimensions that both breaks supersymmetry and also gives a tachyon-free spectrum. This result is due to multiple groups of authors [41, 43, 44, 45]. The corresponding nonabelian gauge group is, happily, $SO(16) \times SO(16)$. Thus, the circle compactification of this nonsupersymmetric ground state with the above-mentioned, radius-dependent Wilson line background interpolates smoothly between supersymmetric $E_8 \times E_8$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$ ground states at zero and infinite radius and a nonsupersymmetric, but tachyon-free, $SO(16) \times SO(16)$ heterotic string theory at intermediate values of the radius. This intriguing example of a *spontaneous* breaking of supersymmetry by continuous variations of the string background parameters was explored in both analytic, and numerical, analyses [49, 50], an idea revived recently in [51]. I should emphasize that none of these works made any reference to the physical interpretation of their results in terms of the behavior of the string canonical ensemble at finite temperature.

Spontaneous supersymmetry breaking via Scherk-Schwarz orbifold compactification is an old subject in the string theory literature [46, 47, 52], and conjectural mention of a possible thermal interpretation has been made for certain nonsupersymmetric, $(-1)^F$, $\mathbb{Z}_2$ orbifolds of type II, heterotic, or type I vacua, with target spacetime fermions in a sector with half-integer timelike momentum [47, 52]. We believe this physical interpretation is *incorrect.* First, as pointed out above, since all of these nonsupersymmetric ground states are tachyonic, they are clearly unacceptable as candidates for describing an *equilibrium* string thermodynamics valid in the full temperature range.

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Secondly, it should be noted that such Z_2 orbifold compactifications meet a familiar low energy finite temperature field theory constraint: namely, that massless spacetime fermions should acquire a tree-level $1/\beta$ dependent mass relative to the massless bosons [56], by introducing a half-integer shift in the momenta for spacetime fermions relative to that for spacetime bosons. In contrast, our prescription for thermal mode number dependent phases described in section 3.3 achieves the same low energy constraint *without* invoking any half-integer shifts in thermal momentum. In other words, our prescription for thermal phases holds in the ordinary circle-compactified theory and is, therefore, consistent with Lorentz invariance of the ten-dimensional string vacuum.[3]

What is the significance of the radius (inverse temperature) dependent Wilson line background in the low energy gauge-gravity field theory limit? Recall that the appropriate gauge choice for the quantization of finite temperature gauge theory is axial gauge, $A_0 = 0$, which correctly accounts for the requisite physical degrees of freedom in the vector potential. Of course, this argument does not preclude a modified gauge choice $A_0 = \beta^{-1}$, with A_0 set equal to a temperature dependent constant. In string theory, of course, the inverse temperature both parameterizes the Wilson line, and also corresponds to the vev of a scalar field, with corresponding vertex operator ∂X^0, where X^0 is Euclidean embedding time. Thus, the canonical ensemble at fixed temperature corresponds to a particular string background, and we have a one-parameter family of Wilson line backgrounds described by the interpolating vacuum energy density at generic β. We emphasize that the choice of Wilson line is determined uniquely by the requirement that the string vacuum functional transform correctly under a thermal duality transformation, interchanging supersymmetric $E_8 \times E_8$ and $\mathrm{Spin}(32)/Z_2$ ground states in the zero, and infinite, inverse temperature limits. But, happily, the Wilson line also results in a thermal spectrum that is *tachyon-free* at all temperatures starting from zero, shifting the masses of the would-be thermal tachyons above the massless threshold. Finally, as we will see below, this choice of Wilson line also results in a modular invariant expression for the one-loop vacuum functional.

In summary, it is possible to formulate an equilibrium string thermodynamics whose low energy limit corresponds to the equilibrium modes of the canonical ensemble in a finite temperature 10d gauge-gravity quantum field theory. In this paper, we verify in section 5.1 that the high temperature growth in the vacuum energy density as inferred from the low energy limit of our expression, where the contribution of massive string modes has been suppressed, indeed reproduces the familiar T^{10} growth of quantum field theory. This strongly holographic behavior was originally conjectured by Atick and Witten in 1988 [2], and shown to be a consequence of thermal duality in the closed bosonic string ensemble by Polchinski more recently [32]. It is reassuring that this startling behavior for the canonical ensemble at temperatures above the string mass scale is nevertheless compatible with the T^{10} growth of finite temperature field theory reproducible in the low energy limit.

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[3]Note that the expression for the one-loop vacuum energy density given in Eq. (3.125) of the first of the references in [47, 51] describes an unstable nonsupersymmetric type IB vacuum, with a closed string tachyon contributing to the torus amplitude. RR tadpole cancellation occurs precisely as in the supersymmetric ground state. In contrast with our expression given in Eq. (32), supersymmetry is broken in the one-loop vacuum amplitude by introducing half-integer momenta for spacetime fermionic states, relative to integer momenta for spacetime bosons, in the torus, Klein bottle, and Mobius strip graphs. Although these are interesting string theory alternatives to traditional scenarios for dynamical supersymmetry breaking, they do not bear any relation whatsoever to the thermal vacuum, a point clarified by consideration of the target spacetime low energy physics. I thank E. Dudas for clarification [48].

Let us now clarify a technical issue that has been a major source of confusion in the literature on the string canonical ensemble: the choice of boundary conditions on the worldsheet fermions in the one-loop superstring path integral at finite temperature. This was discussed at length in Atick and Witten's work [2], regretfully, with an incorrect conclusion that violates modular invariance. As is well-known, it is the norm in d-dimensional finite temperature field theory, where $d > 2$, to impose antiperiodic boundary conditions on the fermions in the direction of Euclidean time. However, we should remind the reader that *there is no spin-statistics theorem in two dimensions* [42]. Thus, while it is important that the low energy limit of the finite temperature string theory exhibit the familiar properties expected from a finite temperature 10D quantum field theory, such considerations apply to the *target spacetime* fermions, rather than the Ramond-Neveu-Schwarz (RNS) worldsheet fermions. In particular, we are free to choose the spin structure on the worldsheet in any way we please, subject only to the string theory consistency conditions [42]: super-Weyl and super-diffeomorphism invariance of the Polyakov path integral [1, 33], including, in particular, modular invariance of the one-loop vacuum functional in the case of an oriented closed string theory. We will find that modular invariance forces us to include sectors with both aperiodic and periodic boundary conditions on the worldsheet fermions in the closed orientable type II ensemble.

Consider the possibility of introducing supersymmetry breaking phases in the string path integral that are thermal mode number dependent, an idea proposed by Atick and Witten in [2]. Such phases will, in fact, be present in the expressions for the one-loop string free energy of the general type IIA or type IIB thermal vacua with Ramond-Ramond sector Dbranes described in later sections of this paper, although we will motivate them from different considerations: by requiring self-consistency with the physics of the low energy limit, a finite temperature gauge theory living on the worldvolume of the Dbranes. Since there is, in fact, no spin-statistics theorem in two dimensions, the ad-hoc prescription given by Atick and Witten in section 5.2 of their paper for the choice of thermal mode number dependent phases has no justification; notice that no derivation or proof has been offered. Unfortunately, a serious problem develops if one introduces a phase factor $(-1)^n$, where n is thermal momentum mode number, in the one-loop closed string path integral: such a phase violates modular invariance. Ref. [2] present an argument in sec 5.1 that correlates the (0,0), (n,0), (0,w), and (n,w) soliton sectors with, respectively, phase factors: 1, $(-1)^n$, $(-1)^w$, $(-1)^{n+w}$. The resulting expression in Eq. (5.20) of their paper for the free energy of the type II oriented closed string ensemble, which we have reproduced for clarity in Eq. (24) of section 34.1, violates modular invariance in the (n,0) and (n,w) sectors. The corrected modular invariant expression for the one-loop free energy of the pure closed orientable type II string ensemble appears in the same section.

We will now explain how the choice of supersymmetry breaking thermal mode number dependent phases in the string path integral can, instead, be determined by requiring self-consistency with the physics of the low energy limit of the string amplitude. Matching to some well-established property of the field theory expected in the low energy limit of the string theory puts constraints on the asymptotics of the one-loop string path integral and, hence, on any unknown phases present in it. Such low energy constraints can often be invoked to determine unambiguously all of the, a priori, undetermined phases in the string path integral, as emphasized by us in [74, 33]. Consider the tadpole and anomaly free unoriented 10D type IIB open and closed string theory with 32 D9branes, or its T-dual 10D type IIA vacuum with 32 D8branes, at finite temperature. It is clear that the usual cancellations between spacetime fermions and spacetime bosons, mass level by mass level, due

to target spacetime supersymmetry must not hold except in the zero temperature limit. We can introduce supersymmetry breaking phase factors, $(-1)^n$, in the type IB vacuum functional, where n is thermal momentum mode number, or $(-1)^w$, where w is thermal winding mode number, in the dual type I$'$ vacuum functional [3]. As shown in section 4.3, we will take care to insert an *identical* phase factor for the contributions to the one-loop vacuum functional from worldsheets with each of the following one-loop topologies, annulus, Mobius strip, and Klein bottle, in order to preserve at finite temperatures the familiar mechanism for the cancellation of the tadpole for the unphysical Ramond-Ramond scalar that holds in the supersymmetric, zero temperature, ground state. The contribution from pure closed oriented one-loop graphs with the topology of a torus vanishes, as is explained in detail in section 4.2, and in [3, 5]. Let us examine the low energy physics in the thermal type IB vacuum. It is well known that expansion of the one-loop modular integral as a function of worldsheet modulus, $q=e^{-2\pi t}$, in the limit $t\to\infty$, enables one to identify the number of spacetime fermions and spacetime bosons at each mass level in the open string mass spectrum. Recall that in the supersymmetric zero temperature vacuum, the absence of a tachyon *required* a relative minus sign between the contributions of the (AP,AP) and (AP,P) spin structure sectors, where AP and P denote, respectively, aperiodic and periodic boundary conditions on the worldsheet (RNS) fermions, implying that there is no contribution to the one-loop vacuum energy at order $q^{-1/2}$ [32, 74, 33]. As shown in section 4.3, we can preserve this property in the finite temperature type IB vacuum by a judicious choice of phases, for every value of thermal momentum mode number, n, *thereby ensuring the complete absence of thermal tachyons.*

As in the heterotic string, there is a temperature dependent Wilson line contributing an overall shift to the vacuum energy in the finite temperature vacuum. At low temperatures, the dominant contribution to the type IB vacuum energy is from the n=1 thermal momentum mode. The $(-1)^n$ phase multiplying the (AP,AP) and (AP,P) sectors has been chosen so that there are no longer any spacetime fermions contributing at the n=1 level. Thus, *spacetime supersymmetry is spontaneously broken by thermal effects precisely in the manner expected from generic d-dimensional, $d > 2$, finite temperature quantum field theory considerations* [56]: the massless spacetime bosons of the zero temperature vacuum do not acquire a tree-level mass, while the massless spacetime fermions of the zero temperature vacuum have acquired an $O(1/\beta)$ tree-level mass. Notice that, as expected, the spacetime bosons will acquire a mass at the next order in string perturbation theory, which can be computed by considering the appropriate one-loop two point function of massless spacetime boson vertex operators.[4] We should emphasize that the remaining relative phases in the expression for the free energy, which is given in Eq. (32), and the dependence on N, the number of D9branes, are uniquely determined by the requirement of tadpole cancellation for the unphysical RR scalar in the absence of target space supersymmetry. Our choice of phases thus ensures that the usual argument for RR scalar tadpole cancellation in the zero temperature vacuum [32] goes thru unchanged in the finite temperature expression. This is important, since the RR unphysical scalar tadpole cancellation is BPS physics, having nothing to do whatsoever with thermal effects on the type IB vacuum. Interestingly, it can also be verified that the dilaton tadpole is also absent as a consequence of requiring the absence of the unphysical RR sector tadpole. Finally, notice that the expression in Eq.

---

[4]We remind the reader that the one-loop vacuum amplitude in string theory yields the tree level mass spectrum, where the radiative corrections to the mass formula have not been taken into account. The radiative corrections can be obtained by computing the appropriate two-point function of vertex operators in string loop perturbation theory.

(32) for the vacuum functional interpolates smoothly between the supersymmetric zero temperature limit, with gauge group $O(32)$, and the finite temperature result with gauge group $O(16){\times}O(16)$. Remarkably, we have found a nonsupersymmetric type IB vacuum with supersymmetry broken by thermal effects, but without the appearance of a dilaton tadpole. Further details, and equations, can be found in section 4.3.

We should also note that Atick and Witten [2] make an incorrect, and unfortunate, assertion about thermal duality: it is indeed reasonable that the free energy of any sensible physical system should be a monotonically increasing function of temperature, but there is nothing "unphysical" about the generating functional of connected vacuum graphs being thermal self-dual. The Polyakov path integral yields $W(\beta)$, *not* $F(\beta)$, and $W(\beta)$ is indeed thermal self-dual in the case of the closed bosonic string theory [1]. The free energy of the string ensemble is given by $F(\beta){=}{-}W(\beta)/\beta$, which is, happily, a monotonically increasing function of temperature. The authors then go on to dismiss the significance of the thermal duality properties of the heterotic string vacuum functional. With the exception of the work by Brandenberger and Vafa [28], the notion of thermal duality has played an increasingly diminishing role in the discussions of string thermodynamics that have followed Atick and Witten's influential paper. There has been some investigation of evidence for the long string transition in the context of a microcanonical ensemble [25, 28, 29]. More recently, there have been investigations of the ensemble of type II strings with Dbrane sources in both the canonical, and microcanonical, approaches [53], but *without* an explicit expression for the one-loop free energy of the canonical ensemble. Regretfully, and without exception, these works have instead focussed on what we have pointed out above is a serious physical misinterpretation, namely, the idea that the first of the tachyonic winding mode instabilities might be interpreted as a signal for a Hagedorn phase transition. We reiterate that this physical interpretation is anyhow *untenable*: the thermal spectrum deduced from the expressions for the one-loop free energy that appear in all of these previous works is already replete with tachyonic momentum modes at all temperatures starting from zero.

Our starting point in this paper is the generating functional of connected one-loop vacuum string graphs, $W(\beta) \equiv \ln \mathrm{Z}(\beta)$, derived from first principles in the Polyakov path integral formalism following [6, 1, 33].[5] The vacuum energy density can, of course, be directly inferred from $W(\beta)$.

---

[5]In earlier papers that include discussion of the bosonic string ensemble as a pedagogical toy model [4, 3], we pointed out that, a priori, there is a peculiar ambiguity in the Euclidean time prescription for finite temperature quantum theories that can be illustrated by a canonical ensemble of strings: unlike point particles, strings are one-dimensional objects sensitive to the topology of the embedding space. In physics applications where Lorentz invariance of the target spacetime is not such a sacrosanct principle it appears to us, a priori, an open question whether Euclidean time in the thermal prescription has the topology of a circle, or that of an interval. However, to avoid the misconception that any of our results for the canonical ensemble of the Lorentz invariant 10D supersymmetric string theories rests on such ambiguity, we have removed all mention of the issue in this paper. We make the standard identification of inverse temperature with the radius of the Euclidean timelike coordinate: $\beta{=}2\pi r_{\mathrm{circ.}}$, which has the topology of a circle. But this identification is unambiguously mandated by the relationship of a given thermal background of string theory with Euclidean timelike embedding coordinate, via Wick rotation, to a corresponding Lorentz invariant background of string theory. It should be noted that a spontaneous violation of Lorentz invariance can indeed occur in string/M theory, but this is only in a string vacuum with nontrivial background field, such as an antisymmetric two-form potential $B_{\mu\nu}$ [54]. To avoid confusion, no such background fields will be introduced in the Lorentz invariant 10D flat spacetime string theory backgrounds whose finite temperature behavior is discussed in this paper.

Let us recall the basic thermodynamic identities of the canonical ensemble [10]:

$$F = -W/\beta = V\rho, \quad P = -\left(\frac{\partial F}{\partial V}\right)_T, \quad U = T^2\left(\frac{\partial W}{\partial T}\right)_V, \quad S = -\left(\frac{\partial F}{\partial T}\right)_V, \quad C_V = T\left(\frac{\partial S}{\partial T}\right)_V.$$
(3)

Note that $W(\beta)$ is an intensive thermodynamic variable without explicit dependence on the spatial volume. F is the Helmholtz free energy of the ensemble of free strings, U is the internal energy, and ρ is the finite temperature effective potential, or vacuum energy density at finite temperature. S and C_V are, respectively, the entropy and specific heat of the thermal ensemble. The pressure of the string ensemble simply equals the negative of the vacuum energy density, as is true for a cosmological constant, just as in an ideal fluid with negative pressure [11, 10]. The enthalpy, $H=U+PV$, the Helmholtz free energy, $F=U-TS$, and the Gibbs function, also known as the Gibbs free energy, is $G=U-TS+PV$. As a result of these relations, *all* of the thermodynamic potentials of the string ensemble have been give a simple, first-principles, formulation in terms of the path integral over worldsheets. Notice, in particular, that since $P=-\rho$, the one-loop contribution to the Gibbs free energy of the string ensemble vanishes identically! Finally, the reader may wonder why we omit mention of *tree*-level contributions to the vacuum energy density. In the case of pure closed string theories, or for the closed string sector of the type I and type I′ open and closed string theories, there are none [32]: the underlying reason can be traced to reparameterization invariance of worldsheets with the topology of a sphere. On the other hand, in the presence of Dbranes and orientifold planes, one indeed finds nontrivial tree-level terms in the vacuum energy density contributed by disk, and crosscap, amplitudes [32, 33]. However, in the R-R sector tadpole-free vacuum described in sections 4.3 and 5, the overall tree-level contribution to the vacuum energy density vanishes [32].

We have emphasized that many of the significant new results in this paper: the T^2 growth of the string free energy, the rigorous demonstration of a duality phase transition in the Kosterlitz-Thouless universality class in the heterotic string, and the evidence for a novel high temperature long string phase in the open and closed string theories, are consequences of the thermal duality transformation properties required of the expression for the string vacuum functional at finite temperature. We should mention that a completely different notion of thermal duality was extensively explored in the early days of string theory, notably by E. Alvarez and collaborators [14].[6] Duality also makes an appearance in [21, 22, 23].[7]

The notion of a holographic principle has played an active role in recent discussions of quantum gravity and conjectures for M theory [77]. We should emphasize that the T^2 growth of the string free energy at high temperatures demonstrated in this paper is a much more drastic reduction in the

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[6]These authors also attempt to "derive" the properties of the microcanonical ensemble from those of the canonical ensemble [14, 29], or vice versa, applying them to models for superstring cosmology. We have already noted that such attempts are conceptually flawed. A technical reason why the inverse Laplace transform linking the microcanonical and canonical closed string ensembles cannot be carried out in closed form is that the integration over the moduli of the Riemann surfaces summed in the expression for the closed string free energy cannot be carried out explicitly.

[7]A much more conjectural application of *thermal self-duality*, based on certain mathematical identities satisfied by integrals over the fundamental domain of the modular group of the torus, appears in recent works by Dienes and Lenneck [60]. Since thermal self-duality is being required of all of the thermodynamic potentials, the standard relationships in Statistical Mechanics linking energy and entropy, or energy and specific heat, for example, no longer hold. Thus, it is unclear how to interpret the thermal self-dual expressions computed in [60] in physical terms.

growth in the number of high temperature degrees of freedom than is suggested by the holographic principle: $F \propto T^2$ for the $d = 10$ string ensemble, *not* simply as fast as an area, namely, $\propto T^{d-1}$, rather than as a volume, $\propto T^d$, as conjectured in the quantum gravity setting for a holographic d-dimensional fundamental theory [77].

3 Canonical Ensemble of Heterotic Closed Strings

The heterotic closed string theory is possibly the closest supersymmetric string analog of the closed bosonic string theory considered by Polchinski in [1], so let us begin with this case, as in [3]. Consider the ten-dimensional supersymmetric $E_8 \times E_8$ theory at zero temperature. The $\alpha' \to 0$ low energy field theory limit is 10D $N=1$ supergravity coupled to $E_8 \times E_8$ Yang-Mills gauge fields. What happens to the supersymmetric ground state of this theory at finite temperature? Assuming that a stable thermal ensemble exists, the finite temperature heterotic ground state with nine noncompact spatial dimensions is expected to be tachyon-free, while breaking supersymmetry. Moreover, consistency with the low energy limit, which is a finite temperature gauge-gravity theory, implies that the thermal string spectrum must contain Matsubara-like thermal momentum modes. But the thermal spectrum is also likely to contain winding modes as expected in a closed string theory [1]. Most importantly, since we are looking for a self-consistent string ground state with good infrared and ultraviolet behaviour, it is important that the one-loop vacuum functional preserve the usual worldsheet symmetries of $(1,0)$ superconformal invariance and one-loop modular invariance. Finally, all of our considerations are required to be self-consistent with thermal duality transformation defined as a Euclidean timelike T-Duality transformation. Since the action of spatial target space dualities on the different supersymmetric string theories are extremely well-established, the finite temperature vacuum functional is required to interpolate between the following two spacetime supersymmetric limits: in the $\beta \to \infty$ limit we recover the vacuum functional of the supersymmetric $E_8 \times E_8$ heterotic string, while in the $\beta = 0$ limit we must recover, instead, the vacuum functional of the supersymmetric $\mathrm{Spin}(32)/Z_2$ heterotic string. The reason is that the $E_8 \times E_8$ and $\mathrm{Spin}(32)/Z_2$ heterotic string theories are related by the Euclidean timelike T-duality transformation: $\beta_{E_8 \times E_8} \to \beta_{\mathrm{Spin}(32)/Z_2} = 4\pi^2 \alpha'/\beta_{E_8 \times E_8}$.

3.1 Axial Gauge and the Euclidean Timelike Wilson Line

The generating functional of connected one-loop vacuum string graphs is given by the expression:

$$W_{\text{het.}} = L^9 (4\pi^2 \alpha')^{-9/2} \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2^2} (2\pi\tau_2)^{-9/2} |\eta(\tau)|^{-14} Z_{\text{het.}}(\beta) \quad . \tag{4}$$

The canonical ensemble of oriented closed strings occupies the box-regularized spatial volume L^9. The thermodynamic limit is approached as follows: we take the limit $\alpha' \to 0$, $L \to \infty$, holding the dimensionless combination, $L^9 (4\pi^2 \alpha')^{-9/2}$, fixed. The function $Z_{\text{het.}}(\beta)$ contains the contributions from the rank $(17,1)$ Lorentzian self-dual lattice characterizing this particular ground state of the circle compactified $E_8 \times E_8$ heterotic string. Thus, we wish to identify a suitable interpolating expression for W_{het} valid at generic values of β, matching smoothly with the known vacuum functional

of the supersymmetric $E_8 \times E_8$ string theory at zero temperature ($\beta = \infty$):

$$W_{\text{het.}}|_{T=0} = \quad L^{10}(4\pi^2\alpha')^{-5} \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2^2}(2\pi\tau_2)^{-5} \cdot \frac{1}{|\eta(\tau)|^{16}} \cdot \left[\left(\frac{\bar{\Theta}_3}{\bar{\eta}}\right)^4 - \left(\frac{\bar{\Theta}_4}{\bar{\eta}}\right)^4 - \left(\frac{\bar{\Theta}_2}{\bar{\eta}}\right)^4\right]$$
$$\times \left[\left(\frac{\Theta_3}{\eta}\right)^8 + \left(\frac{\Theta_4}{\eta}\right)^8 + \left(\frac{\Theta_2}{\eta}\right)^8\right]^2 \quad . \tag{5}$$

Thus, $Z_{\text{het.}}(\beta)$ describes the thermal mass spectrum of $E_8 \times E_8$ strings.

It turns out that the desired result already exists in the heterotic string literature. The modular invariant possibilities for the sum over spin structures in the 10d heterotic string have been classified, both by free fermion and by orbifold techniques [41, 44, 43, 45], and there is a *unique* nonsupersymmetric and tachyon-free solution with gauge group $SO(16) \times SO(16)$. Recall the radius-dependent Wilson line background described by Ginsparg in [40] which provides the smooth interpolation between the heterotic $E_8 \times E_8$ and $SO(32)$ theories in nine dimensions. We have: $\mathbf{A} = \frac{2}{x}(1, 0^7, -1, 0^7)$, $x = (\frac{2}{\alpha'})^{1/2}r_{\text{circ.}}$. Introducing this background connects smoothly the 9D $SO(16) \times SO(16)$ string at generic radii with the supersymmetric 10d limit where the gauge group is enhanced to $E_8 \times E_8$. Note that the states in the spinor lattices of $SO(16) \times SO(16)$ correspond to massless vector bosons only in the noncompact limit. Generically, the $(17, 1)$-dimensional heterotic momentum lattice takes the form $E_8 \oplus E_8 \oplus U$. Here, U is the $(1, 1)$ momentum lattice corresponding to compactification on a circle of radius $r_{\text{circ.}} = x(\alpha'/2)^{1/2}$. A generic Wilson line corresponds to a lattice boost as follows [40]:

$$(\mathbf{p}; l_L, l_R) \rightarrow (\mathbf{p}'; l'_L, l'_R) = (\mathbf{p} + wx\mathbf{A}; u_L - \mathbf{p} \cdot \mathbf{A} - \frac{wx}{2}\mathbf{A} \cdot \mathbf{A}, u_R - \mathbf{p} \cdot \mathbf{A} - \frac{wx}{2}\mathbf{A} \cdot \mathbf{A}) \quad . \tag{6}$$

$\mathbf{p}$ is a 16-dimensional lattice vector in $E_8 \oplus E_8$. As shown in [40], the vacuum functional of the supersymmetric 9d heterotic string, with generic radius *and* generic Wilson line in the compact spatial direction, can be written in terms of a sum over vectors in the boosted lattice:

$$W_{\text{SS}}(r_{\text{circ.}}; \mathbf{A}) = \quad L^{10}(4\pi^2\alpha')^{-5} \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2^2}(2\pi\tau_2)^{-5}|\eta(\tau)|^{-16}\frac{1}{8}\frac{1}{\bar{\eta}^4}(\bar{\Theta}_3^4 - \bar{\Theta}_4^4 - \bar{\Theta}_2^4)$$
$$\times \left[\frac{1}{\eta^{16}}\sum_{(\mathbf{p}'; l'_L, l'_R)} q^{\frac{1}{2}(\mathbf{p}'^2 + l'^2_L)}\bar{q}^{\frac{1}{2}l'^2_R}\right] \quad . \tag{7}$$

W_{SS} describes the supersymmetric heterotic string with gauge group $SO(16) \times SO(16)$ at generic radius. The partition function of the nonsupersymmetric but tachyon-free 9d string with gauge group $SO(16) \times SO(16)$ at generic radii is given by [44, 50, 41, 43, 45, 49]):

$$Z_{\text{NS}}(r_{\text{circ.}}) = \frac{1}{4}\left[(\frac{\Theta_2}{\eta})^8(\frac{\Theta_4}{\eta})^8(\frac{\bar{\Theta}_3}{\eta})^4 - (\frac{\Theta_2}{\eta})^8(\frac{\Theta_3}{\eta})^8(\frac{\bar{\Theta}_4}{\eta})^4 - (\frac{\Theta_3}{\eta})^8(\frac{\Theta_4}{\eta})^8(\frac{\bar{\Theta}_2}{\eta})^4\right]\sum_{n,w} q^{\frac{1}{2}l^2_L}\bar{q}^{\frac{1}{2}l^2_R} \quad . \tag{8}$$

However, by identifying an appropriate interpolating function as in previous sections and appropriate background field, we can continuously connect this background to the supersymmetric $E_8 \times E_8$ string.

Since $x=(\frac{2}{\alpha'})^{1/2}\frac{\beta}{2\pi}$, from the viewpoint of the low-energy finite temperature gauge theory the timelike Wilson line is simply understood as imposing a modified axial gauge condition: A^0=const. The dependence of the constant on background temperature has been chosen to provide a shift in the mass formula that precisely cancels the contribution from low temperature $(n,0)$ tachyonic modes. As before, we begin by identifying an appropriate modular invariant interpolating function:

$$
\begin{aligned}
Z_{\text{het.}} = \quad & \frac{1}{2}\sum_{n,w}\left[(\frac{\Theta_2}{\eta})^8(\frac{\Theta_4}{\eta})^8(\frac{\bar{\Theta}_3}{\eta})^4 - (\frac{\Theta_2}{\eta})^8(\frac{\Theta_3}{\eta})^8(\frac{\bar{\Theta}_4}{\eta})^4 - (\frac{\Theta_3}{\eta})^8(\frac{\Theta_4}{\eta})^8(\frac{\bar{\Theta}_2}{\eta})^4\right]q^{\frac{1}{2}l_L^2}\bar{q}^{\frac{1}{2}l_R^2}\\
& \frac{1}{4}\sum_{n,w}e^{\pi i(n+2w)}\left[(\frac{\bar{\Theta}_3}{\eta})^4 - (\frac{\bar{\Theta}_2}{\eta})^4 - (\frac{\bar{\Theta}_4}{\eta})^4\right]\left[(\frac{\Theta_3}{\eta})^{16} + (\frac{\Theta_4}{\eta})^{16} + (\frac{\Theta_2}{\eta})^{16})\right]q^{\frac{1}{2}l_L^2}\bar{q}^{\frac{1}{2}l_R^2}\\
& +\frac{1}{2}\sum_{n,w}e^{\pi i(n+2w)}\{(\frac{\bar{\Theta}_3}{\eta})^4(\frac{\Theta_3}{\eta})^8\left[(\frac{\Theta_4}{\eta})^8 + (\frac{\Theta_2}{\eta})^8\right] - (\frac{\bar{\Theta}_2}{\eta})^4(\frac{\Theta_2}{\eta})^8\left[(\frac{\Theta_3}{\eta})^8 + (\frac{\Theta_4}{\eta})^8\right]\\
& -(\frac{\bar{\Theta}_4}{\eta})^4(\frac{\Theta_4}{\eta})^8\left[(\frac{\Theta_2}{\eta})^8 + (\frac{\Theta_3}{\eta})^8\right]\}q^{\frac{1}{2}l_L^2}\bar{q}^{\frac{1}{2}l_R^2}\quad. \qquad (9)
\end{aligned}
$$

As in previous sections, the first term within square brackets has been chosen as the nonsupersymmetric sum over spin structures for a *chiral* type 0 string. This function appears in the sum over spin structures for the tachyon-free $SO(16){\times}SO(16)$ string given above. Notice that taking the $x{\to}\infty$ limit, by similar manipulations as in the type II case, yields the partition function of the supersymmetric 10D $E_8{\times}E_8$ string.

Consider accompanying the $SO(17,1)$ transformation described above with a lattice boost that decreases the size of the interval [40]:

$$
e^{-\alpha_{00}} = \frac{1}{1 + |\mathbf{A}|^2/4} \quad . \qquad (10)
$$

This recovers the Spin(32)/Z$_2$ theory compactified on an interval of size $2/x$, but with Wilson line $\mathbf{A}=x\text{diag}(1^8,0^8)$ [40]. Thus, taking the large radius limit in the *dual* variable, and with dual Wilson line background, yields instead the spacetime supersymmetric 10D $Spin(32)/Z_2$ heterotic string. It follows that the $E_8{\times}E_8$ and $SO(32)$ heterotic strings share the same tachyon-free finite temperature ground state with gauge symmetry $SO(16){\times}SO(16)$. The Kosterlitz-Thouless transformation at $T_C=1/2\pi\alpha'^{1/2}$ is a continuous thermal phase transition in this theory.

The thermal duality transition in this theory is in the universality class of the Kosterlitz-Thouless transition [30]: namely, the partial derivatives of the free energy to arbitrary order are analytic functions of temperature at either side of the phase boundary. The duality transition interchanges thermal momentum modes of the $E_8{\times}E_8$ theory with winding modes of the $Spin(32)/Z_2$ theory, and vice versa. Note that the vacuum functional, the Helmholtz and Gibbs free energies, the internal energy, and all subsequent thermodynamic potentials, are both finite and tachyon-free.

3.2   Thermal Duality and the High Temperature Degrees of Freedom

In the closed bosonic string theory, the generating functional for connected one-loop vacuum string graphs is invariant under the thermal self-duality transformation: $W(T) = W(T_c^2/T)$, at the string

scale, $T_c = 1/2\pi\alpha'^{1/2}$. As already noted by Polchinski [32], we can infer the following thermal duality relation which holds for both the Helmholtz free energy, $F(T) = -T \cdot W(T)$, and the effective potential, $\rho(T) = -T \cdot W(T)/V$ of the closed bosonic string theory:

$$F(T) = \frac{T^2}{T_C^2} F(\frac{T_C^2}{T}), \qquad \rho(T) = \frac{T^2}{T_C^2} \rho(\frac{T_C^2}{T}) \quad . \tag{11}$$

In the case of the heterotic string, we will show that the thermal duality relation instead relates, respectively, the free energies of the $E_8 \times E_8$ and $\mathrm{Spin}(32)/Z_2$ theories. Since we deal with a supersymmetric string theory, it is convenient to restrict ourselves to the contributions to the vacuum energy density from target space *bosonic* degrees of freedom alone, as was done in [38, 5]:

$$F(T)_{E_8 \times E_8} = \frac{T^2}{T_C^2} F(\frac{T_C^2}{T})_{\mathrm{Spin}(32)/Z_2}, \qquad \rho(T)_{E_8 \times E_8} = \frac{T^2}{T_C^2} \rho(\frac{T_C^2}{T})_{\mathrm{Spin}(32)/Z_2} \quad . \tag{12}$$

Consider the high temperature limit of this expression:

$$\lim_{T \to \infty} \rho(T)_{E_8 \times E_8} = \lim_{T \to \infty} \frac{T^2}{T_C^2} \rho(\frac{T_C^2}{T})_{\mathrm{Spin}(32)/Z_2} = \lim_{(T_C^2/T) \to 0} \frac{T^2}{T_C^2} \rho(\frac{T_C^2}{T})_{\mathrm{Spin}(32)/Z_2} = \frac{T^2}{T_C^2} \rho(0)_{\mathrm{Spin}(32)/Z_2} \quad , \tag{13}$$

where $\rho(0)$ is the contribution to the cosmological constant, or vacuum energy density, at zero temperature from target space bosonic degrees of freedom alone. Note that it is finite. Thus, at high temperatures, the contribution to the free energy of either heterotic theory from target space bosonic degrees of freedom alone grows only as fast as T^2. In other words, the growth in the number of degrees of bosonic freedom at high temperature in the heterotic string ensemble is only as fast as in a *two-dimensional* field theory. This is significantly slower than the T^{10} growth of the high temperature degrees of freedom expected in the ten-dimensional low energy field theory.

Notice that the prefactor, $\rho(0)/T_C^2$, in the high temperature relation is unambiguous, a consequence of the normalizability of the generating functional of one-loop vacuum graphs in string theory [1]. It is also background dependent: it is computable as a continuously varying function of the background fields upon compactification to lower spacetime dimension [49]. The relation in Eq. (13) is unambiguous evidence of the holographic nature of perturbative heterotic string theory: *there is a drastic reduction in the degrees of freedom in string theory at high temperature, a conjecture first made in [2].*

Starting with the duality invariant expression for the string effective action functional, we can derive the thermodynamic potentials of the heterotic string ensemble. The Helmholtz free energy follows from the definition below Eq. (4), and is clearly finite at all temperatures, with no evidence for either divergence or discontinuity. The internal energy of the heterotic ensemble takes the form:

$$
\begin{aligned}
U \equiv \quad & -\left(\frac{\partial W}{\partial \beta}\right)_V \\
= \quad & L^9 (4\pi^2 \alpha')^{-9/2} \frac{1}{2} \int_{\mathcal{F}} \frac{|d\tau|^2}{4\tau_2^2} (2\pi\tau_2)^{-9/2} |\eta(\tau)|^{-16} \frac{4\pi\tau_2}{\beta} \sum_{n,w \in \mathbb{Z}} \left(w^2 x^2 - \frac{n^2}{x^2}\right) \cdot q^{\frac{1}{2}\mathbf{l}_L^2} \bar{q}^{\frac{1}{2}\mathbf{l}_R^2} \cdot Z_{[\mathrm{SO}(16)]^2},
\end{aligned}
\tag{14}
$$

where $Z_{[SO(16)]^2}$ denotes the sums over spin structures appearing in Eq. (9). Notice that $U(\beta)$ vanishes precisely at the string scale, $T_c = 1/2\pi\alpha'^{1/2}$, $x_c = \sqrt{2}$, where the internal energy contributed by winding sectors cancels that contributed by momentum sectors. Note also that the internal energy changes sign at $T = T_C$. Hints of this behaviour are already apparent in the numerical analysis, with plots, of the one-loop effective potential that appears in [49, 50].

It is easy to demonstrate the analyticity of infinitely many thermodynamic potentials in the vicinity of the critical point. It is convenient to define:

$$
\begin{aligned}
[d\tau] \quad &\equiv \tfrac{1}{2} L^9 (4\pi^2\alpha')^{-9/2} \left[\frac{|d\tau|^2}{4\pi\tau_2^2} (2\pi\tau_2)^{-9/2} |\eta(\tau)|^{-16} \cdot Z_{[SO(16)]^2} e^{2\pi i n w\tau_1} \right] \\
y(\tau_2; x) \quad &\equiv 2\pi\tau_2 \left(\frac{n^2}{x^2} + w^2 x^2 \right) \quad .
\end{aligned}
\tag{15}
$$

Denoting the mth partial derivative with respect to β at fixed volume by $W_{(m)}$, $y_{(m)}$, we note that the higher derivatives of the vacuum functional take the simple form:

$$
\begin{aligned}
W_{(1)} &= \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} (-y_{(1)}) \\
W_{(2)} &= \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} (-y_{(2)} + (-y_{(1)})^2) \\
W_{(3)} &= \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} (-y_{(3)} - y_{(1)} y_{(2)} + (-y_{(1)})^3) \\
\cdots &= \cdots \\
W_{(m)} &= \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} (-y_{(m)} - \cdots + (-y_{(1)})^m) \quad .
\end{aligned}
\tag{16}
$$

Referring back to the definition of y, it is easy to see that both the vacuum functional and, consequently, the full set of thermodynamic potentials, are analytic in x. Notice also that third and higher derivatives of y are determined by the momentum modes alone:

$$
y_{(m)} = (-1)^m n^2 \frac{(m+1)!}{x^{m+2}}, \quad m \geq 3 \quad .
\tag{17}
$$

For completeness, we give explicit results for the first few thermodynamic potentials:

$$
F = -\frac{1}{\beta} W_{(0)}, \quad U = -W_{(1)}, \quad S = W_{(0)} - \beta W_{(1)}, \quad C_V = \beta^2 W_{(2)}, \cdots \quad .
\tag{18}
$$

The entropy is given by the expression:

$$
S = \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} \left[1 + 4\pi\tau_2 (-\frac{n^2}{x^2} + w^2 x^2) \right] \quad ,
\tag{19}
$$

For the specific heat at constant volume, we have:

$$
C_V = \sum_{n,w} \int_{\mathcal{F}} [d\tau] e^{-y} \left[16\pi^2\tau_2^2 (-\frac{n^2}{x^2} + w^2 x^2)^2 - 4\pi\tau_2 (3\frac{n^2}{x^2} + w^2 x^2) \right] .
\tag{20}
$$

17

Explicitly, the Helmholtz free energy takes the form:

$$F(\beta) = -\tfrac{1}{2}\frac{1}{\beta}L^9(4\pi^2\alpha')^{-9/2}\int_{\mathcal{F}}\frac{|d\tau|^2}{4\pi\tau_2^2}(2\pi\tau_2)^{-9/2}|\eta(\tau)|^{-16}\left[\sum_{n,w}Z_{[\mathrm{SO}(16)]^2}q^{\frac{1}{2}\frac{\alpha'p_L^2}{2}}\bar{q}^{\frac{1}{2}\frac{\alpha'p_R^2}{2}}\right]\quad, \qquad (21)$$

while for the entropy of the heterotic string ensemble, we have the result:

$$S(\beta) = \quad \tfrac{1}{2}L^9(4\pi^2\alpha')^{-9/2}\int_{\mathcal{F}}\frac{|d\tau|^2}{4\pi\tau_2^2}\frac{(2\pi\tau_2)^{-9/2}}{|\eta(\tau)|^{16}}\sum_{n,w}\left[1+4\pi\tau_2(-\frac{n^2}{x^2}+w^2x^2)\right]Z_{[\mathrm{SO}(16)]^2}q^{\frac{1}{2}\frac{\alpha'p_L^2}{2}}\bar{q}^{\frac{1}{2}\frac{\alpha'p_R^2}{2}}.$$

$$(22)$$

The thermodynamic potentials of the heterotic ensemble are finite normalizable functions at all temperatures starting from zero. In summary, the heterotic string ensemble displays a continuous phase transition at the string scale mapping thermal winding modes of the $E_8{\times}E_8$ theory to thermal momentum modes of the $\mathrm{Spin}(32)/Z_2$ theory, and vice versa, unambiguously identifying a phase transition of the Kosterlitz-Thouless type at $T_C{=}1/2\pi\alpha'^{1/2}$ [30, 4].

4   Type I and Type II Open and Closed String Theories

The Type I and Type II string theories can have both open and closed string sectors, and the vacuum can contain Dbranes: sources for Ramond-Ramond charge, with worldvolume Yang-Mills fields [32, 33]. It is therefore helpful to consider them in a unified treatment. We will begin with the pure oriented closed string sector common to all of these theories.

4.1   Closed Orientable Sector of the Type II String

We begin with a discussion of the pure type II oriented closed string one-loop vacuum functional. A Euclidean T-duality transformation mapping the thermal IIA vacuum to the thermal IIB vacuum simply maps IIA winding to IIB momentum modes, and vice versa. In the absence of a Ramond-Ramond sector, the expression for the normalized generating functional of connected one-loop vacuum graphs takes the form:

$$W_{\mathrm{II}} = L^9(4\pi^2\alpha')^{-9/2}\int_{\mathcal{F}}\frac{d^2\tau}{4\tau_2^2}(2\pi\tau_2)^{-4}|\eta(\tau)|^{-14}Z_{\mathrm{II}}(\beta)\quad, \qquad (23)$$

where the spatial volume $V{=}L^9$, while the dimensionless, or scaled, spatial volume is $L^9(4\pi^2\alpha')^{-9/2}$. Notice that in the $\alpha' \to 0$ limit one can simultaneously take the size of the "box" to infinity while keeping the rescaled volume fixed. This defines the approach to the thermodynamic limit. The function $Z_{\mathrm{II\ orb.}}(\beta)$ is the product of contributions from worldsheet fermions and bosons, $Z_F Z_B$, and is required to smoothly interpolate between finite temperature nonsupersymmetric, and spacetime supersymmetric zero, and infinite, temperature limits. The spectrum of thermal modes will be unambiguously determined by modular invariance. Spacetime supersymmetry breaking will be implemented by the introduction of *phases* in the interpolating function. Such phases can depend on thermal mode number. They must be chosen *compatible with the requirement that spacetime supersymmetry is restored in the zero temperature limit of the interpolating function.*

We should note that thermal mode number dependent phases in the free energy were first proposed by Atick and Witten in [2]. There is, unfortunately, a problem with their particular implementation of this idea that we must point out at the outset. Let us refer the reader to section 5.1 of [2] where the procedure for introducing thermal phases in the string path integral was first discussed. The thermal spectrum of the closed type II and heterotic string theories includes both momentum and winding modes. Since closed strings can wind an integer number of times around the timelike direction, such a solitonic sector of the string path integral *could* be weighted by the factor $(-1)^w$, where w is the winding number: as emphasized in the Introduction, there is no argument based on the spin-statistics theorem alone for two-dimensional RNS fermions that *requires* such a phase factor. Notice that a $(-1)^w$ phase factor leaves the modular invariance properties of the one-loop closed string path integral untouched. But let us first clarify what has been a source of confusion to many readers of our earlier work [3]: what goes wrong with modular invariance in Ref. [2]'s treatment of the type II closed oriented string ensemble, and why do we report a different result for this sector of the type I, or type II, thermal ensemble?

Since there is no spin-statistics theorem in two dimensions, the argument in section 5.2 of [2], thru to the explicit prescription given in Eq. (5.4) for the phase factor in the general (n, w) sector, for every choice of spin structure for the RNS fermions, has no justification. *Notice that there is no proof, or derivation, given for such a choice of phases.* Unfortunately, a serious problem develops if one introduces a phase $(-1)^n$, where n is thermal momentum, in the one-loop closed string path integral: such a phase violates modular invariance. AW present an argument in section 5.1 that correlates the (0,0), (n,0), (0,w), and (n,w) soliton sectors with, respectively, phase factors: 1, $(-1)^n$, $(-1)^w$, $(-1)^{n+w}$. The result for the free energy of the type II oriented closed string ensemble given in Eq. (5.20) of [2] is reproduced here:

$$
\begin{aligned}
\frac{F_1}{VT} \sim \quad & -(4\pi^2\alpha')^{-9/2} \int_F d^2\tau \tau_2^{-6} |\eta(\tau)|^{-24} \sum_{n,w} e^{-S_\beta(n,w)} \\
& \times [(|\Theta_3|^8 + |\Theta_4|^8 + |\Theta_2|^8)(0,\tau) + e^{\pi i(n+w)}(\Theta_2^4\bar{\Theta}_4^4 + \Theta_4^4\bar{\Theta}_2^4)(0,\tau) \\
& - e^{i\pi n}(\Theta_2^4\bar{\Theta}_3^4 + \Theta_3^4\bar{\Theta}_2^4)(0,\tau) - e^{i\pi w}(\Theta_3^4\bar{\Theta}_4^4 + \Theta_4^4\bar{\Theta}_3^4)(0,\tau)] \quad ,
\end{aligned}
\tag{24}
$$

where $S_\beta(n, w)$ is the worldsheet soliton action in the (n, w) sector of the thermal closed oriented string spectrum. As will be shown below, the second and third terms inside the square brackets are *not* modular invariant, as a consequence of the thermal momentum mode number dependence in the phase. For the heterotic string, similar motivational discussion of spin structures weighted by such phases is presented in sections 5.3, 5.4 and 5.7 of [2], but no explicit result for the free energy was given. The authors appear not to have noticed the clash of their prescription for phases with one-loop modular invariance.

To be explicit, let us refer the reader to a standard textbook treatment of the Poisson resummation of the closed string partition function with momentum and winding modes in a compact direction of the target spacetime. Keep in mind that inverse temperature, β, corresponds to $2\pi R$ in the textbook discussion. Introduction of a phase, for example, $(-1)^{n+w}$, would modify the Poisson resummation, the starting point in Eq. 8.2.9 of the text [32], as follows:

$$
|\eta(\tau)|^{-2} \sum_{n,w=-\infty}^{\infty} e^{i\pi(n+w)} \cdot \exp\left[-\pi\tau_2\left(\frac{\alpha'n^2}{R^2} + \frac{w^2R^2}{\alpha'}\right) + 2\pi inw\tau_1\right] \quad ,
\tag{25}
$$

We perform a Poisson resummation of the momentum mode number, namely, n:

$$\sum_{n=-\infty}^{\infty} \exp\left[-\pi a n^2 + 2\pi i b n\right] = a^{-1/2} \cdot \sum_{m=-\infty}^{\infty} \exp\left[-\pi(m-b)^2/a\right] \quad . \tag{26}$$

In the absence of a phase factor, the result will be manifestly invariant under $\tau \to \tau + 1$, if we also let $m \to m + w$. The result would also be invariant under $\tau \to -1/\tau$, if we simultaneously let $m \to -w$, $w \to m$. So, in the absence of a phase, the partition function is modular invariant.

Are there any possible generalizations with nontrivial (n, w) dependent phases? Introduction of the phase factor $(-1)^w$ inside the summation in Eq. (25) does not alter the Poisson resummation on n, giving a modular invariant result. But a phase factor of the form $(-1)^n$, or $(-1)^{n+w}$, where n is the thermal momentum mode number, as in the second and third terms of Atick and Witten's result given in Eq. (24), ruins one-loop modular invariance. Upon including a $(-1)^{n+w}$ phase, the Poisson resummation results in the following additional terms in the exponent:

$$2\pi R Z_X(\tau) \cdot \sum_{m,w=-\infty}^{\infty} e^{i\pi w} \cdot \exp\left[-\pi R^2 \left\{|m - w\tau|^2 + (w\tau_1 - m + \tfrac{1}{4})\right\}/\alpha'\tau_2\right] \quad . \tag{27}$$

Clearly, the extra terms *cannot* be absorbed in a shift on the summation variable m, or w. Nor can they be absorbed in the corresponding modular transform of the products of Jacobi theta functions arising from the RNS worldsheet fermions, since these are w independent. This is the basis for our claim that, by contrast, the expression for the one-loop vacuum functional we will give below not only satisfies modular invariance, but is also the *unique* acceptable solution.

To reiterate, the supersymmetry breaking thermal mode number dependent phases must be chosen compatible with the requirement that spacetime supersymmetry is restored in the zero, and infinite, temperature limits of the interpolating vacuum functional of the type IIA (IIB) string ensemble. The *unique* modular invariant interpolating function satisfying these requirements is:

$$Z_{\mathrm{II}}(\beta) = \frac{1}{2} \frac{1}{\eta\bar\eta} \left[\sum_{w,n\in\mathbb{Z}} q^{\frac{1}{2}(\frac{2\pi\alpha'n}{\beta} + \frac{w\beta}{2\pi\alpha'})^2} \bar{q}^{\frac{1}{2}(\frac{2\pi\alpha'n}{\beta} - \frac{w\beta}{2\pi\alpha'})^2} \right] \{ (|\Theta_3|^8 + |\Theta_4|^8 + |\Theta_2|^8)$$
$$+ e^{\pi i w} [(\Theta_2^4 \bar\Theta_4^4 + \Theta_4^4 \bar\Theta_2^4) - (\Theta_3^4 \bar\Theta_4^4 + \Theta_4^4 \bar\Theta_3^4 + \Theta_3^4 \bar\Theta_2^4 + \Theta_2^4 \bar\Theta_3^4)] \} \quad . \tag{28}$$

The world-sheet fermions have been conveniently complexified into left- and right-moving Weyl fermions. As in the superstring, the spin structures for all ten left- and right-moving fermions, ψ^μ, $\bar\psi^\mu$, $\mu = 0, \cdots, 9$, are determined by those for the world-sheet gravitino associated with left- and right-moving N=1 superconformal invariances.

To understand our result for a modular invariant interpolating function that also captures the correct thermal duality properties, first recall the expression for Z_{SS}— the zero temperature, space-time supersymmetric, limit of our function given by the ordinary GSO projection:

$$Z_{\mathrm{SS}} = \frac{1}{4} \frac{1}{\eta^4 \bar\eta^4} \left[(\Theta_3^4 - \Theta_4^4 - \Theta_2^4)(\bar\Theta_3^4 - \bar\Theta_4^4 - \bar\Theta_2^4) \right] \quad , \tag{29}$$

Notice that the first of the relative signs in each round bracket preserves the tachyon-free condition. The second relative sign determines whether spacetime supersymmetry is preserved in the zero

temperature spectrum [74]. Next, notice that Z_{SS} can be rewritten using theta function identities as follows:

$$
\begin{aligned}
Z_{\mathrm{SS}} \quad &= \{[|\Theta_3|^8 + |\Theta_4|^8 + |\Theta_2|^8] \\
&\quad + [(\Theta_2^4\bar{\Theta}_4^4 + \Theta_4^4\bar{\Theta}_2^4) - (\Theta_3^4\bar{\Theta}_4^4 + \Theta_4^4\bar{\Theta}_3^4 + \Theta_3^4\bar{\Theta}_2^4 + \Theta_2^4\bar{\Theta}_3^4)]\} \quad .
\end{aligned}
\tag{30}
$$

Either of the two expressions within square brackets is modular invariant. The first expression can be recognized as the nonsupersymmetric sum over spin structures for the type 0 string [32]. Under a thermal duality transformation, small β_{IIA} maps to large $\beta_{IIB}=\beta_C^2/\beta_{IIA}$, also interchanging the identification of momentum and winding modes, $(n, w)_{IIA} \rightarrow (n' = w, w' = n)_{IIB}$. Formally, the expression for the vacuum functional of the IIB closed oriented thermal ensemble will take identical mathematical form to that for the IIA closed oriented ensemble.

From the modular invariant expression for the vacuum energy density of the type II string derived in the previous section, we can conclude that the physical Hilbert space of the pure closed orientable thermal type II string contains a tachyonic instability at all temperatures above $T_{w=1}$, the temperature at which the first of the thermal winding modes turns tachyonic. To see this, consider the mass formula in the (NS,NS) sector for world-sheet fermions, with $\mathbf{l}_L^2 = \mathbf{l}_R^2$, and $N=\bar{N}=0$:

$$
(\mathrm{mass})^2_{nw} = \frac{2}{\alpha'}\left[-1 + \frac{2\alpha'\pi^2 n^2}{\beta^2} + \frac{\beta^2 w^2}{8\pi^2\alpha'}\right] \quad .
\tag{31}
$$

This is the only sector that contributes tachyons to the thermal spectrum of the SCFT. The potentially tachyonic physical states are the pure momentum and pure winding states, $(n, 0)$ and $(0, w)$, with $N=\bar{N}=0$. As in the closed bosonic string analysis, we can compute the temperatures below, and beyond, which these modes become tachyonic in the absence of oscillator excitations. Each momentum mode, $(\pm n, 0)$, is tachyonic *upto* some critical temperature, $T_n^2 = 1/2n^2\pi^2\alpha'$, after which it turns marginal (massless). Conversely, each winding mode $(0, \pm w)$, is tachyonic *beyond* some critical temperature, $T_w^2 = w^2/8\pi^2\alpha'$. *Only the pure winding mode tachyons survive the thermal physical state conditions of the type II ensemble.* Thus, at temperatures above $T_{w=1}$, we apparently do not have a viable equilibrium type II oriented closed string thermodynamics. Fortunately, as is shown in the accompanying paper [38, 5], the flow of the worldsheet RG is in a direction towards the noncompact supersymmetric fixed point corresponding to, respectively, the type IIA, or type IIB, vacuum, at zero temperature. A direct corollary of this result will be, as we show in the next section, that the closed orientable sub-sector of generic type I and type II string theories with Dbranes does not break supersymmetry at one-loop order as a consequence of thermal effects: the contribution to the one-loop finite temperature vacuum energy density from the torus vanishes.

4.2 Cancellation of Ramond-Ramond Sector Tadpole

We move on to a discussion of the unoriented sectors of the type IB and type I' open and closed string theories. Consider the one-loop free energy, $F(\beta)$, of a gas of free type IB strings in the presence of the Euclidean timelike Wilson line. F is obtained from the generating functional for connected vacuum string graphs, $F(\beta)=-W(\beta)/\beta$. The one-loop free energy receives contributions from worldsurfaces of four different topologies [32, 33]: torus, annulus, Mobius strip, and Klein

bottle. The torus is the sum over closed oriented worldsheets and the result is, therefore, identical to that derived in section 2 for the type IIB string theory, refer to Eqs. (23) and (28). Notice that the closed orientable string sector of the type IB theory does not distinguish between the Dbrane worldvolume and the bulk spacetime orthogonal to the branes, since the closed oriented strings live in all ten dimensions of spacetime. Nor does this sector have any knowledge about the Yang-Mills sector, or of the Euclidean timelike Wilson line, since the Yang-Mills fields of type IB arise as open string excitations. Thus, the comments we have made earlier regarding worldsheet renormalization group (RG) flows in the type IIA or type IIB vacuum in the absence of a Ramond-Ramond sector apply as well for the closed orientable sub-sector of the generic type IB string theory. Namely, although we begin with an, a priori, non-vanishing contribution from the torus amplitude to the vacuum energy density, the thermal spectrum contains a tachyonic winding mode instability at temperatures above $T_{w=1}$, and the resulting RG flow is in the direction restoring the IR stable supersymmetric closed string fixed point vacuum at zero temperature. In other words, the torus contribution to the vacuum energy density in the generic unoriented open and closed type IB thermal vacuum always vanishes [3], and supersymmetry is *not* broken by thermal effects in this sector of the theory at one-loop order.

The contribution to the vacuum energy density from the remaining three worldsheet topologies that contribute at one loop order is given by the Polyakov path integral summing surfaces with two boundaries, with a boundary and a crosscap, or with two crosscaps [32]. We require a tachyon-free thermal spectrum, namely, a fixed line of thermal type IB vacua parameterized by the inverse temperature β so that the canonical ensemble describes an equilibrium statistical mechanics of type IB strings. Finally, as in the supersymmetric type IB vacuum, consistency requires the absence of a tadpole for the unphysical Ramond-Ramond (R-R) state in the thermal vacuum [8, 69, 32, 5].

On the other hand the usual cancellation between spacetime fermions and spacetime bosons at each mass level due to target spacetime supersymmetry must not hold except in the zero temperature limit. This is in precise analogy with the finite temperature analysis given earlier for closed string theories and is achieved by introducing a phase factor, $(-1)^n$ in the type IB, where n is thermal momentum or type I′, vacuum functional [2]. Note that we insert an identical phase for the contributions to $F(\beta)$ from worldsheets with each of the three remaining classes of one-loop graphs— annulus, Mobius Strip, and Klein Bottle, in order to preserve the form of the RR scalar tadpole cancellation in the zero temperature ground state. The result for the vacuum functional at one-loop order takes the form:

$$W_{\text{IB}}(\beta) = \quad L^9 (4\pi^2 \alpha')^{-9/2} \int_0^\infty \frac{dt}{2t} \frac{(2\pi t)^{-9/2}}{\eta(it)^8} \sum_{n \in \mathbb{Z}} [\ N^2 (Z_{[0]} - e^{\imath \pi n} Z_{[1]})$$
$$+ 2^{10} \left(Z_{[0]} - e^{\imath \pi n} Z_{[1]} \right) \ - \ 2^6 N \left(1 - e^{\imath \pi n} \right) Z_{[2]} \ ] \times q^{4\alpha' \pi^2 n^2/\beta^2}.$$
(32)

The thermal modes of the type IB theory correspond to a Matsubara-like frequency spectrum with timelike momentum spectrum: $p_n = 2n\pi/\beta$, where $n \in \mathbb{Z}$. The functions $Z_{[0]}$, $Z_{[1]}$, and $Z_{[2]}$ are defined as follows:

$$Z_{[0]} = \quad (\frac{\Theta_{00}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{01}(it;0)}{\eta(it)})^4$$

$$Z_{[1]} = \quad (\frac{\Theta_{10}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{11}(it;0)}{\eta(it)})^4$$

$$Z_{[2]} = \quad \left(\frac{\Theta_{01}(it;0)\Theta_{10}(it;0)}{\eta(it)\Theta_{00}(it)}\right)^4 \quad , \tag{33}$$

where the (00), (10), (01), and (11), denote, respectively, (AP,AP), (P,AP), (AP,P), and (P,P), boundary conditions on worldsheet fermions along the two cycles of the strip. In this expression, P and AP denote the periodic and antiperiodic boundary condition on fermions.

Let us explain our choice of phases in W_{IB} in more detail. As is well known, an asymptotic expansion of the Jacobi theta functions in powers of $q=e^{-2\pi t}$ enables one to identify the number of spacetime fermions and spacetime bosons at each mass level in the open string spectrum. Recall that in the supersymmetric zero temperature vacuum, the absence of a tachyon *required* a relative sign between the contributions of the (AP,AP) and (AP,P) spin structure sectors, denoted (00) and (01) above, implying that there was no contribution to the one-loop free energy at $O(q^{-1/2})$ [32, 74, 33]. We will preserve this property in the finite temperature vacuum, and for every value of n, thus ensuring the complete absence of thermal tachyons. This is the reason (AP,AP) and (AP,P) spin structure contributions have been grouped together in the function $Z_{[0]}$ without any *relative* thermal mode number dependent phase.

The next order in the asymptotic expansion, $O(q^0)$, gives the massless open string spectrum in the zero temperature vacuum. The Wilson line contributes an overall shift to the vacuum energy of the finite temperature vacuum. At low temperatures, the dominant contribution to the vacuum energy is from the n=1 thermal momentum mode. The $(-1)^n$ phase multiplying the (10) and (11) sectors has been chosen so that there are no longer any spacetime fermions contributing at the n=1 level. Thus, spacetime supersymmetry is spontaneously broken by thermal effects precisely in the manner expected from generic d-dimensional, $d > 2$, finite temperature quantum field theory considerations: the massless spacetime bosons of the zero temperature vacuum do not acquire a tree-level mass, while the massless spacetime fermions of the zero temperature vacuum have acquired an $O(1/\beta)$ tree-level mass. Notice that, as expected, the spacetime bosons will acquire a mass at the next order in string perturbation theory, which can be computed by considering the appropriate one-loop two point function of massless spacetime boson vertex operators.[8] We should emphasize that the remaining relative phases in Eq. (32), and the dependence on N, the number of D9branes, are uniquely determined by the requirement of tadpole cancellation for the unphysical R-R state even in the absence of target space supersymmetry. Our choice of phases ensures that the usual argument for the cancellation of the tadpole for the unphysical R-R state in the zero temperature vacuum [8, 69, 32, 5] goes thru unchanged in the finite temperature expression. This is important, since an uncancelled tadpole for an unphysical R-R state cannot be removed by an adjustment of the background fields: this would be a genuine inconsistency of the type IB vacuum, whether at zero temperature, or at finite temperature. Interestingly, it can be verified that the dilaton tadpole is also absent in the thermal vacuum as a consequence of our requiring the absence of the R-R sector tadpole. Remarkably, we have found a *nonsupersymmetric* type IB vacuum with supersymmetry

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[8]We remind the reader that the one-loop vacuum amplitude in string theory yields the tree level mass spectrum, where the radiative corrections to the mass formula have not been taken into account. The radiative corrections can be obtained by computing the appropriate two-point function of vertex operators in string loop perturbation theory.

broken by thermal effects, but without the appearance of a dilaton tadpole.

A further check of self-consistency with the low energy gauge theory limit is to verify the expected T^{10} growth of the free energy in the low energy field theory limit. At low temperatures far below the string scale we expect not to excite any thermal modes beyond the lowest lying field theory modes, and should therefore recover the T^{10} growth in the free energy. Thus, as is standard lore in the string theory-low-energy-field theory correspondence [32, 33], we will expose the $t \to \infty$ asymptotics of the modular integral in Eq. (32), keeping only the leading contribution from the massless modes in the open string spectrum:

$$
\begin{aligned}
F = \quad & -\beta^{-1} \lim_{\beta \to \infty} L^9 (4\pi^2 \alpha')^{-9/2} \int_0^\infty \frac{dt}{2t} (2\pi t)^{-9/2} \sum_{n \in \mathbb{Z}} \left[1 + O(e^{-2\pi t}) \right] q^{4\alpha' \pi^2 n^2 / \beta^2} \\
= \quad & -\beta^{-1} \cdot \beta^{-9} \cdot \rho_{\text{low}} \quad ,
\end{aligned}
\tag{34}
$$

where ρ_{low} is a numerical constant independent of temperature.[9] It is reassuring to recover the expected T^{10} growth characteristic of a ten-dimensional finite temperature gauge theory.

Notice that the thermal momentum modes in the expression for the free energy have no winding mode counterparts because of the absence of self-duality in the type IB spectrum. In the Euclidean T-dual type I′ theory, obtained by letting $\beta \to \beta_C^2/\beta$ in the expressions above, the type IB thermal momentum modes are mapped to type I′ thermal winding modes, each wrapping the T-dual Euclidean time coordinate $X^{0\prime}$ [3]. The type IB timelike Wilson line wrapping X^0 is likewise mapped to the T-dual timelike Wilson line, wrapping the T-dual coordinate $X^{0\prime}$, with saddle point action $-\beta^2 (2\pi t)/4\pi^2 \alpha'$. How should one interpret the existence of the Euclidean T-dual description of the thermal ground state of unoriented open and closed type IB strings? We will close by pointing out that its chief utility is in the clearer understanding it provides of the high temperature behavior of the low energy gauge theory limit, where the contributions from massive string modes has been suppressed.

5   High Temperature Transition to the Long String Phase

It is an old piece of string folklore that at high temperatures, or at high energy densities, a microcanonical ensemble of strings at fixed energy will transition into a long string phase. In other words, most of the ensemble energy resides in one, or more, long, closed strings, surrounded by a sea of short open strings. Evidence for such a phase transition has, of course, been of great interest in models proposed for the dynamics of a cosmic string ensemble [26, 68]. In the discussion that follows in section 5.1, we will investigate the high temperature growth in the number of degrees of freedom in the type I string by an asymptotic estimate of the growth in the number of states at high level in the open string mass spectrum. That calculation requires consideration of the $t \to 0$ asymptotics of the integrand in Eq. (32). We will find a T^2 growth in the free energy at high temperatures for the type IB ensemble in section 5.1, precisely analogous to that obtained for the heterotic string ensemble in section 3.2. The dramatic change in the growth in the free energy above the string scale with a much *slower* growth of the free energy at temperatures $T >> T_C$, is our

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[9]The precise normalization is given in the recent paper [80].

second, and more convincing, indication of a plausible thermal phase transition at a temperature of order the string scale. The order parameter for this transition will be identified in section 5.2.

5.1 High Temperature Behavior of the Type I String Ensemble

We begin by verifying the T^2 growth in the vacuum energy density in the thermal type IB vacuum at high temperatures above the string scale by direct inspection of the ultraviolet asymptotics of our expression for the one-loop vacuum functional. As is well-known [32, 33], if we wish to sample the asymptotic behavior at high mass levels of the open string mass spectrum, we need to consider instead the $t\to 0$ asymptotics of the expression for $F_{\mathrm{IB}}(\beta)$. This will yield an asymptotic estimate for the high temperature behavior of the full string ensemble.

A modular transformation on the argument of the theta functions, $t\to 1/t$, puts the integrand in a suitable form for term-by-term expansion in powers of $e^{-1/t}$; this enables term-by-term evaluation of the modular integral. Isolating the $t\to 0$ asymptotics in the standard way [32], we obtain a most unexpected result:

$$
\begin{aligned}
\lim_{\beta\to 0} F_{\mathrm{IB}}(\beta) = \quad & -\lim_{\beta\to 0}\beta^{-1}L^9(8\pi^2\alpha')^{-9/2}\int_0^\infty \frac{dt}{2t}\frac{t^{-1/2}}{\eta(i/t)^8}\sum_{n\in\mathbb{Z}}[\ N^2(Z_{[0]}-e^{1\pi n}Z_{[1]}) \\
& +2^{10}\left(Z_{[0]}-e^{1\pi n}Z_{[1]}\right) \ - \ 2^6 N\left(1-e^{1\pi n}\right)Z_{[2]}\ ]\times q^{4\alpha'\pi^2 n^2/\beta^2} \\
= \quad & -\beta^{-1}\sum_{n\in\mathbb{Z}}\left[\beta^{-1}\cdot\rho_{\mathrm{high}}\right]+\cdots \quad ,
\end{aligned}
\tag{35}
$$

where ρ_{high} is a constant independent of temperature.[10] Thus, the canonical ensemble of type IB strings displays a T^2 growth in the free energy at high temperatures, characteristic of a *two-dimensional* field theory!

It is helpful to verify the corresponding scaling relations for the first few thermodynamic potentials. The internal energy of the canonical ensemble of type IB strings takes the form:

$$
\begin{aligned}
U = \quad & -\left(\frac{\partial W}{\partial\beta}\right)_V \\
= \quad & L^9(4\pi^2\alpha')^{-9/2}\int_0^\infty\frac{dt}{8t}e^{-8\pi^3\alpha't/\beta^2}\frac{(2\pi t)^{-1/2}}{\eta(i/t)^8}\cdot\sum_{n\in\mathbb{Z}}Z_{\mathrm{open}}4\pi t\left[-\frac{4\alpha'\pi^2(1+n^2)}{\beta^3}\right]q^{4\alpha'\pi^2 n^2/\beta^2},
\end{aligned}
\tag{36}
$$

where $Z_{\mathrm{open}}(i/t)$ is the factor in curly brackets in the expression in Eq. (32). It is clear that the internal energy of the full ensemble also scales as β^{-2} at high temperatures. The entropy is given by the expression:

$$
S=-\beta^2\left(\frac{\partial F}{\partial\beta}\right)_V=-\beta^2\left[\beta^{-2}W(\beta)-\beta^{-1}\left(\frac{\partial W}{\partial\beta}\right)_V\right] \quad ,
\tag{37}
$$

---

[10]The precise normalization appears in our recent work [80].

and we infer that it scales at high temperatures as β^{-1}. Finally, since the specific heat at constant volume is given by:

$$C_V = -\beta \left(\frac{\partial S}{\partial \beta} \right)_V \quad , \tag{38}$$

we infer that it also scales as β^{-1} at high temperatures. Corresponding scaling behavior at *low* temperatures, in agreement with the expected result for the low energy field theory limit can be extracted, as in the previous section, by considering instead the $t \to \infty$ asymptotics of these expressions.

5.2 An Order Parameter for the Long String Phase Transition

A plausible order parameter signalling a thermal phase transition in the type I string theory at T_H is suggested by the correspondence with the low energy finite temperature gauge theory limit. It is well known that the order parameter signalling the thermal deconfinement phase transition in a nonabelian gauge theory at high temperatures is the expectation value of a closed timelike Wilson loop [64, 65, 66, 56]. We wish to investigate the possibility of a thermal deconfinement phase transition in the type I theory at a temperature of order the string scale, conjectured to arise in a gas of short open strings, and characterized qualitatively by long string formation in the deconfined high temperature phase [12, 25, 2, 55, 56, 59].

Since the one-loop vacuum energy density in the type IB thermal vacuum displays no non-analyticity, or discontinuities, as a function of temperature, it is natural to look for evidence in a different string amplitude. A natural choice suggested by the correspondence in the low energy limit to a finite temperature Yang-Mills gauge theory, would be the string theory analog of the expectation value of the Wilson-Polyakov-Susskind loop wrapping the Euclidean time direction, namely, the change in the free energy in the thermal vacuum due to the introduction of an external heavy quark, generally taken to be the order parameter for the deconfinement phase transition in finite temperature gauge theory [64, 65, 56].

As mentioned in the introduction, the appropriate starting point is the Polyakov path integral summing surfaces with the topology of an annulus and with boundaries mapped to a pair of fixed curves, $\mathcal{C}_1$, $\mathcal{C}_2$, in the embedding target spacetime, wrapping the Euclidean time coordinate, and with fixed spatial separation, R, can also be computed from first principles using Riemann surface methodology, an extension of the one-loop vacuum amplitude computation due to Polchinski [1]. The amplitude can be interpreted as an off-shell closed string tree propagator, and the result in closed bosonic string theory, but only in the limit that the macroscopic boundaries, $\mathcal{C}_1$, $\mathcal{C}_2$, were point-like, was first obtained by Cohen, Moore, Nelson, and Polchinski [72], and extended to include the limit of *large* macroscopic loop lengths of interest here by myself in collaboration with Yujun Chen and Eric Novak in [73]. The extension to the macroscopic loop amplitude in the type I and type II string theories with Dbranes appears in [74]. We will calculate the pair correlator of a pair of Wilson-Polyakov loops wrapping the Euclidean time coordinate, extracting the low energy gauge theory limit of the resulting expression where the contribution from massive string modes has been suppressed. Notice that in the limit of vanishing spatial separation, $R \to 0$, the amplitude will be dominated by the shortest open strings, namely, the gauge theory modes in the massless open

string spectrum, and *the worldsheet collapses to a single macroscopic Wilson-Polyakov-Susskind loop wound around the Euclidean time coordinate.* Thus, we have a potentially straightforward route in string theory to extract the standard order parameter [64] of the thermal deconfinement transition in the low energy gauge theory limit. We will analyze this limit of our result for its dependence on temperature.

Consider the pair correlation function of a pair of Polyakov-Susskind loops lying within the worldvolume of the D9branes, and with fixed spatial separation R in a direction transverse to compactified Euclidean time, X^0. Recall that the boundaries of the worldsheet are the closed "world-histories" of the open string endpoint, which couples to the gauge fields living on the worldvolume of the Dbranes. The endpoint state is in the fundamental representation of the gauge group. Thus, when its closed worldline is constrained to coincide with a closed timelike loop in the embedding spacetime, the resulting string amplitude has a precise correspondence in the low energy limit to the correlation function of two closed timelike loops representing the spacetime histories of a pair of static, infinitely massive, quarks with fixed spatial separation. Since we wish to probe the high temperature behavior of the low energy gauge theory limit, we should use the Euclidean T-dual type I′ description of the thermal vacuum.

The result for the pair correlator of temporal Wilson-Polyakov loops in the Euclidean T-dual type I′ vacuum, $\mathcal{W}_{I'}^{(2)}$, derived from first principles from an extension of the ordinary Polyakov path integral in the references [72, 73, 74, 33], takes the remarkably simple form [3]:

$$\mathcal{W}_{I'}^{(2)}(R,\beta) = \lim_{\beta \to 0} \int_0^\infty dt \frac{e^{-R^2 t/2\pi\alpha'}}{\eta(it)^8} \sum_{n \in \mathbb{Z}} q^{n^2\beta^2/4\pi^2\alpha'}$$
$$\times [\ (\frac{\Theta_{00}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{10}(it;0)}{\eta(it)})^4$$
$$- e^{1\pi n} \{ (\frac{\Theta_{01}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{11}(it;0)}{\eta(it)})^4 \}\ ]. \tag{39}$$

The summation variable, n, labels closed string winding modes, each of which wraps around the Euclidean timelike coordinate $X^{0'}$. The static heavy quark potential can be extracted as follows. We set $\mathcal{W}_{I'}^{(2)} = \lim_{s \to \infty} \int_{-s}^{+s} ds V[R,\beta]$, inverting this relation to express $V[R,\beta]$ as an integral over the modular parameter t [71, 32]. The variable s parameterizes proper time for the pointlike infinitely massive static quarks. Consider the q expansion of the integrand valid for $t \to \infty$, where the shortest open strings dominate the modular integral. Retaining the leading terms in the q expansion and performing explicit term-by-term integration over the worldsheet modulus, t, [73, 74, 33], isolates the following interaction potential [3]:

$$V(R,\beta)|_{\beta << \beta_C} = (8\pi^2\alpha')^{-1/2} \int_0^\infty dt e^{-R^2 t/2\pi\alpha'} t^{1/2} \sum_{n \in \mathbb{Z}} (16 - 16e^{i\pi n}) q^{n^2\beta^2/4\alpha'\pi^2} + \cdots$$
$$\simeq \frac{1}{R^3 (1 + \frac{\beta^2}{R^2})^{3/2}} + \cdots \tag{40}$$

where we have dropped all but the contribution from the $n=1$ thermal mode in the last step. At high temperatures, with $\beta << \beta_C$, we can expand the denominator in a power series. The leading temperature dependent correction to the inverse cubic power law is, therefore, $O(\beta^2/R^5)$. At low

temperatures, we must instead use the thermal dual type IB description in order to extract the low energy gauge theory. We have:

$$\mathcal{W}_{\text{IB}}^{(2)}(R, \beta) = \lim_{\beta \to \infty} \int_0^\infty dt \frac{e^{-R^2 t/2\pi\alpha'}}{\eta(it)^8} \sum_{n \in \mathbb{Z}} q^{4\pi^2 \alpha' n^2/\beta^2}$$
$$\times [\ (\frac{\Theta_{00}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{10}(it;0)}{\eta(it)})^4$$
$$-e^{\imath\pi n}\{(\frac{\Theta_{01}(it;0)}{\eta(it)})^4 - (\frac{\Theta_{11}(it;0)}{\eta(it)})^4\} \ ] \quad , \tag{41}$$

from which we can infer:

$$V(R, \beta)|_{\beta >> \beta_C} = (8\pi^2\alpha')^{-1/2} \int_0^\infty dt e^{-R^2 t/2\pi\alpha'} t^{1/2}$$
$$\times \sum_{n \in \mathbb{Z}} (16 - 16e^{i\pi n}) q^{4\alpha'\pi^2 n^2/\beta^2} + \cdots$$
$$\simeq \frac{1}{R^3(1 + \frac{16\pi^4\alpha'^2}{R^2\beta^2})^{3/2}} + \cdots \tag{42}$$

Thus, the low temperature corrections are, instead, a power series in $\alpha'^2/\beta^2 R^2$! In other words, when we use the appropriate representation of the correlator in the temperature regime either below, or above, the string scale transition temperature, we find qualitatively distinct temperature dependent corrections to the leading inverse cubic power law static potential. We can interpret this result as evidence for a phase transition in the low energy gauge theory limit of the type IB string theory. Remarkably, precise computations can be carried out on either side of the phase boundary by utilizing, respectively, the low energy gauge theory limits of a *pair* of thermal dual string theories, type IB and type I′.

We now make an important observation: since the expressions given above are analytic as a function of R, the spatial separation of the loops, we can smoothly take the limit $R \to 0$ where the worldsheet collapses to a single loop. Thus, the single Wilson-Polyakov-Susskind loop expectation value in the finite temperature supersymmetric gauge theory transitions from a $O(1/T^2)$ fall at low temperatures to a T^2 growth at high temperatures above the string scale. From the perspective of finite temperature $SU(N)$ gauge theory, we would have intuitively expected an infinite cost in the free energy to produce a single external quark in the vacuum in the confining regime.[11] However, as explained in [1, 69, 33, 5, 33], whenever a mass parameter in supergravity, or gauge theory, is extracted from the low energy field theory limit of a covariant supersymmetric string theory amplitude, all such mass terms will be unambiguously normalized in units of the fundamental string mass scale, $\alpha'^{1/2}$. This is a consequence of the unique choice of regulator permitted by the worldsheet gauge symmetries: super-diffeomorphism × super-Weyl invariance. The dependence on

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[11]Notice that, unlike the expressions for the string free energy and other thermodynamic potentials in preceding sections of this paper which are well-defined at all temperatures starting from zero, the macroscopic loop amplitude in string theory is not well-defined for Wilson loops at the degenerate limit points, $T = 0$ and $\beta = 0$, since the Riemann surface representation *assumes* a worldsheet with closed boundary loops. Physically, this is simply the statement that the Polyakov-Susskind loop expectation value is not a good order parameter for the deconfining phase transition at zero temperature because the loop is topologically trivial in that limit [64].

temperature of the Polyakov-Susskind loop expectation value on either side of the phase boundary is therefore a very reasonable result from this perspective.[12] A more complete discussion of the numerical significance of this computation will be saved for future work.

As an aside, it is straightforward to include in this result the dependence on an external twoform potential or electromagnetic field. From the previous works [71, 73, 74, 33, 3], the effective string scale in the presence of an external twoform field strength is further *lowered*. The expression above for the potential can be modified by the simple replacement: $2\pi\alpha'^{1/2} \rightarrow 2\pi\alpha'^{1/2}u$, where $\mathcal{F}^{09} = \tanh^{-1}u$ is the electric field strength. Notice that the phase transition temperature would, therefore, be modified by the factor u^{-1} from the zero field prediction. As mentioned in Footnote 5, we do not wish to confuse the reader by introducing two-form background fields that lead to a spontaneous breaking of target space Lorentz invariance, so we will simply omit the corresponding equations. Related discussion can be found in [75].

6   Conclusions

We have shown that each of the six supersymmetric string theories: heterotic, type I, and type II, admits stable thermal backgrounds in which we can formulate an equilibrium statistical mechanics of strings in the canonical ensemble. We have explained in the Introduction, and in Section 2, why preceding attempts [20, 15, 24, 23, 2, 29, 28, 53, 62] to formulate an equilibrium statistical mechanics with a tachyon-free supersymmetric string canonical ensemble have failed: these works did not correctly incorporate the Euclidean T-duality transformations linking the thermal vacua of the supersymmetric string theories in pairs. In particular, the widespread belief that the canonical ensemble of supersymmetric strings breaks down beyond the limiting Hagedorn temperature, T_H, turns out to be *incorrect*.

In the case of the type IIA and type IIB superstring theories, we have shown that a viable thermal vacuum *requires* the introduction of Dbranes carrying Ramond-Ramond charge, and consequently, an open string sector with Yang-Mills gauge fields in the low energy field theory limit. Our analysis holds at one-loop order in the perturbation expansion in the string coupling constant g_s. We remind the reader that the one loop contribution to the vacuum energy density is not accompanied by explicit powers of g_s and our conclusions should, therefore, dovetail neatly with any fully nonperturbative string/M theory analysis that follows in the future. But we have also exhibited some tantalizing properties such as the absence of the dilaton tadpole, and a vanishing vacuum energy density, at one-loop order in the *nonsupersymmetric* unoriented open and closed type IB thermal vacuum with anomaly-free gauge group $O(16) \times O(16)$. This is a remarkable result from the perspective of proposals for spontaneous supersymmetry breaking in string theory [5, 47]. We believe there is sound physical motivation here for a more careful consideration of the string loop corrections to our results in Section 4. Fortunately, there have been some new advances in the understanding of type II superstring loop amplitudes at higher genus in recent years [76], and we defer further discussion to future work.

Next, we have shown that the growth of the vacuum energy density of the string ensemble at

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[12]The precise normalization appears in our recent work [80].

high temperatures far above the string scale is only as fast as that in a *two-dimensional* quantum field theory. This high temperature growth holds for all six supersymmetric string theories. In the case of the type I unoriented open and closed superstring where the low energy gauge theory limit can be readily extracted in closed analytic form, it was reassuring to find in section 4.3 that our expression for the free energy of the string ensemble also reproduces the expected T^{10} quantum field theoretic growth at temperatures much below the string scale, where the contribution from massive string modes has been suppressed. In section 3.2, we have established the existence of a duality phase transition in the Kosterlitz-Thouless universality class mapping the finite temperature ground state of the $E_8 \times E_8$ heterotic string to its $\mathrm{Spin}(32)/\mathbb{Z}_2$ Euclidean T-dual.

Most importantly, the evidence we have given in Section 5 in favor of a novel long string phase at high temperatures in the microcanonical ensemble of short open strings in the type IB string theory is strongly deserving of further investigation, especially as motivated by models for the dynamics of a cosmic string network [26, 25, 29, 68]. The question of interest here is establishing concrete evidence for the *scaling regime*, dominated by one, or more, long winding mode cosmic strings in a bath of thermal radiation described by the short loop length limit of a microcanonical ensemble of open and closed strings. It would also be interesting in this context to extend our analysis for the anomaly-free type IB vacuum with 32 D9branes to an anomaly-free type IB thermal vacuum with N additional D9brane-anti-D9brane pairs [57, 58]. We should note that D-Dbar annihilation has played an important role in recent proposals for generating a suitable inflationary potential in models for early universe superstring cosmology [68]. A further motivation comes from finite temperature large N supersymmetric gauge theory [64, 56, 55, 62, 59]. It would be extremely interesting to explore the low energy gauge theory limit in the *large N limit* of the anomaly-free type IB string theories with $32+N$ D9branes and N anti D9branes constructed in [57, 58]; the one-loop string vacuum amplitude for finite N is given explicitly in the first reference.

We will close with mention of two important insights that apply more broadly to future developments in String/M Theory. First, we should remind the reader that perturbative string theory as formulated in the worldsheet formalism is inherently background dependent: the "heat-bath" representing the embedding target space of fixed spatial volume and inverse temperature is forced upon us, together with any external background fields characterizing the spacetime geometry. Thus, we are ordinarily restricted to the canonical ensemble of statistical mechanics. We should caution the reader that while an immense, and largely conjectural, literature exists on proposals for microcanonical ensembles of weakly-coupled strings [14, 24, 25, 28, 29], the conceptual basis of these treatments is full of holes. Some of the pitfalls have been described in [64, 2]. It should be kept in mind that, strictly speaking, the microcanonical ensemble is what is called for when discussing quantum cosmology, or the statistical mechanics of the Universe [7]: the Universe is, by definition, an isolated closed system, and it is meaningless to invoke the canonical ensemble of the "fundamental" degrees of freedom. However, there are many simpler questions in both early Universe cosmology [26, 28, 67, 53, 62, 68], and in the high temperature behavior of low energy supersymmetric gauge theories [12, 64, 65, 56, 59, 62], that are indeed approachable within the framework of statistical mechanics in the canonical ensemble, such as our discussion of the long string transition in Section 5. In this context, it would be interesting to explore the properties of the microcanonical ensemble of weakly-coupled type IB-I$'$ unoriented strings described in Section 5 using some of the methodology outlined in [24, 25, 29, 28, 53, 62]. The key difference here is that the one-loop

free energy of type I strings *vanishes* as a consequence of RR sector tadpole cancellation. Thus, there is no room for thermal back-reaction and the fixed energy microcanonical ensemble should be well-defined [29].

A second important insight from our analysis of the thermal vacua of the different supersymmetric string theories is that the 10D type II superstring backgrounds with trivial Ramond-Ramond sector, corresponding to pure supergravity theories with 32 supercharges in the low energy limit, are a somewhat artificial truncation of the more generic type II superstrings with Dbranes. These are theories with 16, or fewer, supercharges, and they have both super-Yang-Mills fields and supergravity in their low energy limit, in common with generic backgrounds of the type I and heterotic string theories. Recovering the 10D backgrounds with 32 supercharges as special limits characterized by an *enhanced supersymmetry* in a theory where the generic backgrounds contain both supergravity and super-Yang-Mills sectors and, consequently, 16 or fewer supercharges, is a key element of the matrix proposal for a fundamental theory of emergent spacetime under development in [78, 5, 79]. We comment that such a formalism places the embedding spacetime geometry, and the quantized degrees of freedom within it, on equal footing, thereby implementing the full spirit of Einstein's vision for a fundamental theory. It is also a viable starting point for a discussion of the quantum statistical mechanics of an isolated closed system like our Universe.

Note Added (Sep 2005): This document has far too much material to be readable, and I have separated the useful content into several follow-up papers. The historical account in section 2 may be useful for some readers, except that it should be noted that thermal mode number dependent phases in the type II string vacuum amplitude do not work: it is not possible to spontaneously break spacetime supersymmetry compatible with modular invariance. The conclusion that there is no viable type II canonical ensemble in the absence of a Yang-Mills sector stands. A better discussion of the type II orientifold is given in follow-up work (hep-th/0506143).

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